Subject ST2

CMP Upgrade 2017/18

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus objectives, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2017 CMP to make it suitable for study for the 2018 exams. It includes replacement pages and additional pages where appropriate. Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our 2018 *Student Brochure* for more details.

This CMP Upgrade contains:

- all changes to the Syllabus objectives
- all significant changes to the Core Reading, ActEd Course Notes, Series X Assignments and Question and Answer Bank that will make them suitable for the 2018 exams.

1 Changes to the Syllabus objectives

This section contains all the ***non-trivial*** changes to the Syllabus objectives.

Objective (k) has been updated so that it now reads:

(k) Demonstrate methods of determining the cost of guarantees and options.

- Describe the use of stochastic simulation and the use of option prices to determine the cost of an investment guarantee.

- Describe the assessment of the cost of simple mortality options.

2 Changes to the Core Reading and ActEd Course Notes

This section contains the most ***significant*** changes to the Core Reading and ActEd text. However, if you wish to have all the changes to the ActEd Course Notes you will need to buy a full replacement set of the up-to-date version (which you can do at a significantly reduced price if you have previously bought the full price Course Notes / CMP in this subject).

Chapter 20

Page 17

The second paragraph has been updated as follows:

The concept of non-separability refers to situations where if two events happen together, the combined impact is worse than if they had happened separately. For example, consider longevity risk and expense risk for annuities. If annuitants live longer than expected *and* per policy expenses are higher than expected, there is an additional combined impact due to having these higher than expected expenses payable for longer than expected. If you considered the two risk factors separately, you wouldn't be allowing for this additional combined effect.

Chapter 23

A number of changes have been made throughout the chapter. Section 2.3 has been completely rewritten, *eg* the conventional and North American methods are no longer covered. Replacement pages are attached.

3 Changes to the Q&A Bank

The most significant changes to the Q&A Bank questions and solutions are listed below. However, if you wish to see the fully amended versions, you will need to buy a new CMP (at a significantly reduced price), as indicated at the start of this upgrade.

Q&A Bank Part 4

Question 4.15 has been replaced with a new question. Replacement pages are attached.

Q&A Bank Part 5

Question 5.7 has been replaced with a new question. Replacement pages are attached.

Question 5.17 has been updated so that it no longer refers to the North American method. The solution is unchanged.

The first bullet point in the question now reads:

- AM92 Ultimate (assumed for normal premiums, and for all lives while not taking the option, and for non-takers after the option date)

The following has been added at the end of the question:

You may ignore expenses and the cost of setting up reserves.

Question 5.18 has been deleted.

Q&A Bank Part 7

For Question 7.1, part (i) is an updated version of the original question and part (ii) is new. Replacement pages are attached.

Question 7.6 has been deleted.

For Question 7.7, part (i) has been updated as follows:

(i) Discuss the risks of the proposed product design to the company. [11]

The solution to Question 7.7 (i) has been updated so that the section on capital requirements now reads:

Capital requirements

These could be extremely onerous because the unit fund will exceed the asset share for a considerable period. [1]

There would then be a risk of selling too many policies and endangering solvency. [½]

A new part (iii) has been added to Question 7.7 as follows:

(iii) Discuss whether a non-unit reserve would be held at any point during the term of this contract and comment on its size. [10]

[Total 28]

Replacement pages are attached for the solution to part (iii).

4 Changes to the X assignments

As with the Q&A Bank, we have updated questions and solutions for the changes in the Core Reading and ActEd text.

We only accept the current version of assignments for marking, ie those published for the sessions leading to the 2018 exams. If you wish to submit your script for marking but have only an old version, then you can order the current assignments free of charge if you have purchased the same assignments in the same subject the previous year (ie sessions leading to the 2017 exams), and have purchased marking for the 2018 session.

The most *significant* changes to the assignment questions or solutions are listed below:

X4.5

The first paragraph on page 9 in the solution has been corrected as follows:

A typical bad case could be a ten-year conventional with- or without-profits single premium endowment policy, where reserves could increase by around 20%, from considering

The second paragraph on page 10 in the solution has been updated as follows:

So depending on the mix of business, and the other factors mentioned above, solvency requirements would increase for most companies by anything from 5–10% up to 30–35%. [½]

X5.1

New parts (iv) and (v) have been added. Replacement pages are attached for the question and solution.

X5.2

This question has been replaced by a new question. Replacement pages are attached for the question and solution.

X5.3

This question has been deleted.

5 Other tuition services

In addition to this CMP Upgrade you might find the following services helpful.

5.1 Study material

We offer the following study material in Subject ST2:

- Mock Exam A
- Additional Mock Pack
- ASET (ActEd Solutions with Exam Technique) and Mini-ASET
- MyTest
- Sound Revision
- Revision Notes
- Flashcards
- Online Classroom.

For further details on ActEd's study materials, please refer to the 2018 *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

5.2 Tutorials

We offer the following tutorials in Subject ST2:

- a set of Regular Tutorials (lasting three full days)
- a Block Tutorial (lasting three full days)
- a Revision Day (lasting one full day).

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **www.ActEd.co.uk**.

5.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to the 2018 *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

6 Feedback on the study material

ActEd is always pleased to get feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course please send them by email to **ST2@bpp.com**.

Chapter 23

Cost of guarantees and options

Syllabus objective:

(k) Demonstrate methods of determining the cost of guarantees and options.

- *Describe the use of stochastic simulation and the use of option prices to determine the cost of an investment guarantee.*
- *Describe the assessment of the cost of simple mortality options.*

0 Introduction

In this chapter we shall consider investment guarantees and mortality options. Examples of investment guarantees are found in both unit-linked and non-linked business. Guarantees may relate to surrender and/or maturity values. Surrender guarantees apply at more than one point in time and hence have the additional uncertainty of when they may be taken up.

There is a chance that the asset share underlying the policy does not reach the level guaranteed on payout. Another way of looking at this is that guarantees may constrain the investment policy that an insurance company would otherwise follow. Hence they carry a cost, and so the guarantee should be priced appropriately. We will look at some ways in which the costs can be evaluated.

We will also consider mortality options. These are situations where the policyholder can choose to extend the term or increase the level of cover at normal premium rates but without providing further medical evidence. To the extent that the option might be exercised by someone in poor health, the insurance company will bear a cost: the difference between the ordinary premium rate granted under the terms of the option and that which would have been granted had the life been underwritten.

1 Investment guarantees

1.1 Examples

Traditional life insurance contracts transfer the mortality, expense and investment risk from the policyholder to the life insurance company.

Investment-linked (including unit-linked) contracts leave the investment risk with the policyholder. The attractions of these policies can be enhanced if part of the investment risk is transferred to the insurance company by the contract including, for example:

- **a guaranteed minimum maturity value** (in money terms)
- **a guaranteed minimum surrender value** (again, in money terms)
- **a guaranteed annuity option** (such as described for conventional policies below)**.**

With-profits contracts transfer investment risk to the policyholder, but generally to a lesser extent than for unit-linked contracts (due to smoothing). They typically have some level of guaranteed benefit, which is equivalent to an investment guarantee, *eg* the basic sum assured plus attaching bonus for conventional with-profits business under the additions to benefits approach. Each of the above investment guarantees may be available on with-profits as well as on investment-linked contracts.

The attractions of traditional policies can also be enhanced if the company provides guarantees of investment performance. For example, an option could be provided to convert the maturity value of a without-profits endowment assurance into an immediate annuity on guaranteed terms.

For example, the policyholder takes the cash maturity value but can choose to buy an immediate annuity from the company at a rate guaranteed at policy inception (perhaps 30 years earlier). This will be an attractive option if the policyholder is worried that annuities may become more expensive when he or she retires.

In each case the policyholder is protected from a "downside" risk.

1.2 Implications for the insurance company

The risk assumed by the life insurance company is that at specified times in the future the "backing" assets will be insufficient to meet the guarantees.

This risk is particularly difficult to control if the policyholder has a choice over whether to exercise the option. For example, a guaranteed annuity option gives the policyholder a choice of taking cash or an income at maturity. The company will not know which option the policyholder will choose and hence which option it should match in investment terms.

Additionally, if the guarantee relates to the surrender value, the company will not know the specified time at which assets must cover the liability (*ie* the surrender value) as the time of exercise will be under the control of the policyholder.

Question 23.1

Suppose that an insurance company offers guaranteed annuity payments on its without-profits deferred annuity contract, but invests to meet the open market cash option. Is it at risk from low or high interest rates at retirement?

If the company has control over the investment policy, (*eg* traditional contracts with guarantees), there is conflict between investing to meet the guarantees and investing for maximum performance.

In other words if the company invests the assets backing a with-profits contract to meet a minimum guarantee, then all the policyholder will receive is this minimum return. The policyholder will miss out on the prospect of any out-performance and might just as well have bought a without-profits contract.

If the company chooses not to invest to match the guarantees, it must include the cost of the guarantee in the original pricing basis. If the company has no control over the investment policy, (*eg* unit-linked endowment assurance), it must include the cost of the guarantee in the original charges to the extent that the guarantee will not be matched.

Whether the company mismatches through choice or necessity it will need extra funds from increased premiums or charges in order to offset its risk.

The risk to the company will depend on the outstanding term of the policy. In general, the longer the timeframe involved then the greater the chance that things might go wrong compared with our current forecasts. For instance, offering a guaranteed annuity rate for five years' time will be less risky than offering the same rate for twenty years' time.

However, the picture is more complex for conventional with-profits business. Here, a long timeframe may give the company time to act on profit distribution levels in the event of *eg* disappointing investment returns. The company's management has much less scope for manoeuvre with a short-term policy or a policy that is approaching maturity. If things go wrong there is much less time for corrective action, *eg* reducing reversionary bonuses, to take effect. For conventional with-profits policies, the higher the amounts of *regular* distributions of profit made (like reversionary bonus), the more onerous the guarantee becomes.

Question 23.2

Suppose that an insurance company offers a guaranteed surrender value that can be exercised on an annual basis. Why can't the company match the guarantee by assuming (on the basis of past experience) that a certain percentage of policyholders surrender each year, *ie* why can't it just hold cash for these amounts and hold the remainder in appropriate assets to meet the maturity or death benefits payable to the rest of the policyholders?

1.3 Valuing an investment guarantee

The liability created by an investment guarantee is the excess of the guaranteed amount (*eg* guaranteed maturity value of an endowment, or the fund needed to purchase the "guaranteed annuity" at current market rates), over the cost that would have been incurred at the time in the absence of the guarantee. The policyholder should only choose to exercise the option to take up this guarantee if it is in-the-money (which can alternatively be expressed as "if it bites"), *ie* is financially advantageous.

As discussed earlier, such guarantee costs will be the result of the insurance company being unable or unwilling to invest the premiums to match the guarantee and subsequently losing out as a result of its mismatched investment strategy. The insurance company will need to honour its guarantee whenever this situation occurs and hence must hold sufficient funds to meet its commitments. It will therefore seek to charge an additional premium to meet the liabilities created by the guarantee.

The value of these liabilities can be determined using:

- **option-pricing techniques**
- **stochastic simulation of investment performance.**

For example, in order to be sure of meeting a guarantee to pay a sum of 1,000 increased in line with cash returns, an insurance company would need to invest in cash. However, investing in cash may not be allowable under the terms of the contract, *eg* a unit-linked contract, or it might reduce unacceptably the return that the company could provide to the policyholder. The company therefore needs to charge an extra premium to reflect the extra sums it *may* need to pay out under the guarantee.

There are two main ways in which this extra premium can be assessed:

- Option-pricing/market valuation techniques. These assess the extra premium by looking at the market price of a derivative that the insurance company could acquire to mitigate its risk.
- Stochastic simulation. The extra sums likely to be needed under the guarantee can be modelled by simulating a range of investment scenarios.

Use of option-prices/market valuation techniques

The options incorporated into life insurance contracts are analogous to options traded in the market place. A guaranteed minimum maturity value corresponds to a (European style) put option on the investment funds at an exercise price corresponding to the maturity guarantee.

Suppose that a unit-linked endowment assurance with a single premium of 1,000 has a guaranteed minimum maturity value of 1,200. Suppose also that at maturity, the assets underlying the policy are worth 1,150.

Then at maturity the office will have to sell the underlying assets in the marketplace for 1,150, and then pay 1,200 to the policyholder. Compare this with the situation where the office had granted the policyholder a European style put option on those assets with an exercise price of 1,200, *ie* the office had given the policyholder the right to sell the underlying assets to the office for 1,200 on the maturity date. The office could then resell them for 1,150, ending up in exactly the same position as before (*ie* with a loss of 50).

Hence, if a put option exists in the market for the same (or similar) terms, then the market price for that option provides the company with an estimate of the expected present value of the guarantee under the policy.

A guaranteed minimum surrender value corresponds to a similar American style option or a series of options with different exercise prices which match the guaranteed surrender values.

If a policyholder is given a guaranteed minimum surrender value, then the policyholder may have the option to receive the guaranteed surrender value at any time. So the situation is very similar to the maturity guarantee described above, except that the policyholder can exercise the option whenever he or she wishes to do so. This is equivalent to the office granting the policyholder an American put option, which can be exercised at any time up until the maturity date. A European style option can be exercised only at the maturity date itself.

A guaranteed annuity rate corresponds to a call option on the bonds that would be necessary to ensure the guarantee was met, *ie* at an exercise price which generates the required fixed rate of return. Alternatively it can be mirrored by an option to swap floating rate returns at the option date for fixed rate returns sufficient to meet the guaranteed annuity option (a "swaption").

Recall that a call option is the option to buy an asset at a future date at the exercise price.

Question 23.3

Explain how the insurance company in Question 23.1 could protect itself from the annuity rate guarantee by:

(i) purchasing call options on bonds, or

(ii) purchasing interest rate swap options.

Question 23.4

Suppose the insurance company in Question 23.1 has instead invested to match the future annuity option, and holds bonds. What kind of derivative would the company now look at in order to assess the cost of the option it is providing under its contract?

It is difficult to ensure that the whole investment fund corresponds to a single option traded in the market. However, an approximation is possible by using options written on market indices for equities and bonds.

At the date of policy issue all guarantees will normally be expected to be out-of-the-money, *ie* they will have no intrinsic value because current market rates are more than sufficient to meet the guarantees. However, they will have a time value which is the result of the views of many investors ("the market") of the present value of the likely future costs of the option.

Although current market rates of interest are sufficient to meet the guarantee, a number of investors are sufficiently pessimistic about the prospect of future adverse rate changes to be prepared to bet that the guarantee may bite at some date in the future. The amounts they will pay for this "bet" determines the present value of the option – and this is what we use to estimate our expected present value.

Thus the market price of a suitable option produces a way of costing a financial option or guarantee incorporated in a life insurance policy.

You must be quite clear what is going on here: we are talking about how we can use the current market prices of traded "guarantees" to provide a market-based valuation of the guarantees we may be providing under our insurance policies. (This is actually just an example of a market-consistent valuation, as described in Section 2 of Chapter 20.) We are *not* implying that these derivatives have to be or are *purchased* by the company: this is an entirely separate matter, which would depend on the company's investment strategy in this regard.

It is possible that a guarantee will not be out-of-the-money. For example, current yields might be so low that a life insurance company would be happy to provide a guarantee at a future date based on a higher yield.

For example, we might be providing a guaranteed annuity option at retirement (which may be in many years' time) guaranteeing a yield of, say, 3% *pa*. Current yields might be lower than this (so the guarantee is in-the-money at the present time), but the company might still offer the guarantee if there is an extremely high chance of it being out-of-the-money by the time of retirement.

Stochastic simulation

By projecting forward the value of the assets using a stochastic investment model and comparing this with the sum payable under the guarantee, the insurance company can measure the extent to which additional costs will be incurred under a range of investment scenarios.

A stochastic model of rates of return on investments is used to simulate the future price of assets. The assumptions underlying the model must be carefully evaluated to ensure that they correspond to the company's planned investment strategy. A large number of simulations is needed in order to obtain reliable estimates.

Key assumptions will be the probability distribution used to model the investment return and the mean and variance. Typically 5,000-10,000 simulations might be used.

Example

A company is issuing a five-year unit-linked savings bond with a single premium of 1,000 and guaranteed maturity value equal to the premium rolled up at 3.5% interest *pa*. The unit-linked fund is to be invested in cash.

How could we cost the guarantee using stochastic techniques?

One approach is as follows.

We need to model the returns earned by the cash in the unit-linked fund, so we need to decide on how to model future interest rate movements. In this example, we have assumed that each year's interest rate is equal to the previous year's rate, but with a random drift which is normally distributed with standard deviation of 1% (and mean drift zero). We kick off with current rates programmed in at 5%.

Suppose that we have a random number generator, which is able to generate random numbers that are uniformly distributed in the range 0 to 1. (You may have a function like this on your calculator.) We start by generating say 5 numbers from this distribution:

0.115 0.023 0.864 0.212 0.816

We now use the inverse distribution function method to find the corresponding values from the standard normal distribution:

$$z_1 = \Phi^{-1}(0.115) = -1.2 \qquad z_2 = \Phi^{-1}(0.023) = -2.0$$
$$z_3 = \Phi^{-1}(0.864) = +1.1 \qquad z_4 = \Phi^{-1}(0.212) = -0.8$$
$$z_5 = \Phi^{-1}(0.816) = +0.9$$

Our random interest rate in Year t (I_t say) can now be simulated using:

$$I_t = I_{t-1} + 0.01Z_t$$

where Z_t is the standard normal random variable we have just simulated above.

Now, noting that $I_0 = 0.05$, we get the relevant simulated interest rate in each of the following years:

$$I_1 = 0.05 - 0.012 = 0.038 \qquad I_2 = 0.038 - 0.02 = 0.018$$

$$I_3 = 0.018 + 0.011 = 0.029 \qquad I_4 = 0.029 - 0.008 = 0.021$$

$$I_5 = 0.021 + 0.009 = 0.03$$

This will give a run of interest rates for the next five years. We then repeat the process, say 10,000 times. For each run, we can calculate the projected unit fund of the policy by accumulating the premium, less charges, at the simulated rates of return. For this example we have taken initial expense charges of 50 and ignored any other deductions from the unit fund.

We can then compare the unit fund against the guaranteed minimum maturity value of 1,000 accumulated at 3½% *pa* (*ie* 1,188).

Doing say 10,000 runs we can then look at the results:

Run	***Interest year 1***	***i yr 2***	***i yr 3***	***i yr 4***	***i yr 5***	***Unit fund***	***Guarantee***	***Cost***
1	3.8%	1.8%	2.9%	2.1%	3.0%	1,086	1,188	102
2	7.5%	7.0%	6.8%	5.2%	4.7%	1,285	1,188	0
3	5.1%	6.3%	6.9%	6.4%	7.4%	1,297	1,188	0
4	4.2%	4.9%	4.7%	4.6%	3.5%	1,177	1,188	11
…	…	…	…	…	…	…	…	…
9,999	5.0%	3.9%	3.5%	3.4%	5.4%	1,169	1,188	19
10,000	5.6%	5.7%	7.2%	7.1%	6.1%	1,292	1,188	0
Total	–	–	–	–	–	–	–	142,184

The total costs for the 10,000 runs here amount to 142,000, giving an average result of 14.2 per run. So the cost of the guarantee, which we need to load into the premium, is 14.2.

The principle of stochastic modelling is that the simulated average cost is an estimate of the expected value (of the cost).

Question 23.5

Why is it only an estimate?

For some guarantees the liability if the guarantee is taken is fixed, *eg* maturity guarantee, and for some guarantees the liability will depend on future market conditions, *eg* guaranteed annuity rates. In the latter case, factors influencing the value of the liabilities as well as assets will need to be simulated.

So we will need to project the market bond yields as at the retirement date, in each simulation, which will allow us to compute the cost of the annuity liabilities in retirement. This simulation should be consistent with the assumed performance of the assets over the same timeframe.

If the guarantee relates to a with-profits policy, the modelling will also need to include assumptions about the level of future profit declarations. The assumptions for each simulation need to be consistent with the simulated asset performance.

The company will need to make assumptions about future rates of exercising options, which would take into account expected policyholder behaviour and the size of the guaranteed amount relative to the alternative benefit (*eg* asset share).

So, for example, a guaranteed minimum surrender value on a unit-linked policy is more likely to be taken when the guaranteed amount is greater than the alternative benefit based on the value of the units. Similarly, a with-profits policy is more likely to be surrendered if the guaranteed amount is greater than an alternative benefit based on the asset share. However, it would be quite simplistic to assume a policy would always be surrendered in these situations. Policyholders might value the maturity benefits more highly or expect the guarantee to become even more valuable at a later date.

The present value of the liability can be determined by discounting the simulated cost of exercising the option at a suitable rate. Repeated simulation will generate the probability distribution of the present value of the cost of the option. The company can then charge a premium having a present value which reflects the "market cost" of providing that guarantee; this could be the expected (*ie* average) simulated cost plus a margin.

Question 23.6

In the earlier example, the guarantee applied only at maturity. If the company offered the same 3½% guarantee for surrenders at any time over the five years to maturity, how would you expect the cost to compare with that of the maturity guarantee?

Question 23.7

If the guarantee applied to surrenders at the end of each year, how could we modify the method of the above example to cost the guarantee?

Question 23.8

If the policy were conventional without-profits rather than unit-linked, how would we modify the method to set an appropriate premium?

2 Mortality options

2.1 Examples

It is common for life insurance policies to include options to:

- **purchase additional benefits without providing further evidence of health at the normal premium rates (for a life of that particular age) at the date on which the option is exercised**
- **renew a life insurance policy, *eg* a term assurance at the end of its original term without providing additional evidence of health**
- **change part of the sum assured from one contract to another, *eg* from term assurance to endowment assurance.**

2.2 Implications for the insurance company

The terms and conditions under which the option can be exercised need to be clearly set out in the original policy. Sometimes an option can only be exercised at fixed points of time, *eg* at the end of every five years of a twenty-year term assurance or at anytime providing a qualifying event has occurred, *eg* the birth of a child, taking up a new job at a higher salary. The extent of the option will also be specified, *eg* the additional sum assured cannot exceed the original sum assured.

The terms and conditions for the exercise of the option are designed to reduce "selection against the office", *ie* an excess of lives in poor health using the options to obtain large amounts of life assurance at premium rates that do not reflect their expected mortality.

This is just another example of anti-selection, such as we have already discussed elsewhere in the course.

Question 23.9

Why does restricting the points of time at which the option can be exercised reduce the risk of anti-selection?

The cost of an option is the value of the excess of the premium that should, in the light of full underwriting information, have been charged for the additional assurance over the normal premium rate that is charged. For some lives the option will have no cost.

If a life, who is in good health and who would be expected to satisfy normal underwriting requirements, exercises the option, then the option will generate little or no additional costs. The exercise of the option by lives in poor health will generate considerable additional costs.

The total expected additional costs of an option depend on the health status of those who choose to exercise the option, and the proportion of lives who choose to exercise the option.

In general, the smaller the proportion who exercise the option, the worse will be the subsequent mortality experience of those exercising the option. If a substantial proportion exercise the option, then their subsequent mortality experience will on average be less extreme.

So the cost of a mortality option is (roughly speaking) the product:

$$\{\text{proportion of lives exercising option}\} \times \{\text{average health of lives exercising option}\}$$

Question 23.10

Which is worse to the company: every single person with worse-than-select mortality taking up the option, or if just the small number of very high risk lives take up the option?

Some factors affecting mortality options are:

- The term of the policy with the option. The longer the term, the longer the policyholder will have the option, and the more likely it is that, at some time, his or her health will make the option appear worthwhile.

- The number of times the policyholder gets the chance to exercise the option, *eg* every five years, on every policy anniversary or at any time whatsoever.

- Conditions attaching to exercising the option such as limiting the size of the option or restricting the choice of contracts available under the option.

- The encouragement given to policyholders to exercise the option. As discussed in the Core Reading, if take up of the option is low it tends to be only those who have most to gain who exercise the option. As explained in Question 23.10 above, this can be a good thing, as it could keep the total cost of the option low.

On the other hand, encouraging more of the healthy lives to exercise the option will not cause any additional expected loss, and should contribute to the company's total profit as the company will essentially be issuing lots of new policies to lots of good risks, which should be a profitable proposition.

Publicising the option more widely can achieve greater take up by healthy lives, but care should be taken that the benefit (from future profits) is not outweighed by the risk of attracting a bigger proportion of the loss-making high risk lives from taking up the option as well.

- The extra cost to the policyholder who exercises the option. If the option involves a steep increase in premiums, then the healthier lives might shop around to try to get the same cover cheaper elsewhere. This means that the company will lose out on the potential profit that these policyholders would have generated.

- Selective withdrawals. A healthy life may cancel a ten-year renewable term assurance policy after two years because he or she discovers that the cover without mortality options is much cheaper. The company has not in this case collected the option loading from this person for very long, but is still left with the unhealthy lives who will exercise the option to the cost of the company.

  This is important to understand. Withdrawal by healthy lives before the option date reduces the income (extra premiums) that we would have expected to receive from the whole risk pool. If only healthy lives withdraw, then the cost of the option (which depends only on unhealthy lives) will be little altered. This loss of income therefore constitutes a significant risk in the management of option costs.

Question 23.11

Why might a life company wish to restrict the choice of contracts available under the option?

2.3 Valuing a mortality option

Mortality options are normally valued using cashflow projections. These cashflows would include the additional benefits expected to be payable under the option and the additional premiums expected to be received in relation to these benefits, to the extent to which the option is assumed to be taken up.

The additional premiums would be based on the expected premium rates that would be charged to standard lives for the additional benefit, as at the option exercise date.

So when the policyholder exercises the option he or she will pay exactly the same premium as a new policyholder that has just passed underwriting on that day, *ie* they will be charged a premium based on the select mortality tables in use at that time.

The projections should also allow for any additional expenses incurred in the administration of the option.

For example, expenses would be incurred if the company writes to each policyholder to remind them of the option before the exercise date. Expenses will then be incurred in processing the policyholder's request to exercise the option.

If the purpose of the valuation is pricing the option (rather than setting the liability) then allowance should also be made for the additional reserves that should be held, both before and after exercise.

If we price the option using cashflow projections we will need to calculate the profit vector each year. Recall from Subject CT5 that the profit vector is given by the premiums, less claims, less expenses, plus investment returns, less increase in reserves.

Question 23.12

Explain why a policy that contains a mortality option is likely to need much higher reserves than the equivalent policy without an option.

Valuing a mortality option therefore requires extra assumptions as part of the basis:

- **the probability that the option will be exercised, at each possible exercise date**
- **the additional benefit level that will be chosen, if this is at the discretion of the policyholder**
- **the expected mortality of the lives who choose to exercise the option**
- **the expected mortality of the lives who choose not to exercise the option**
- **additional expenses relating to the option.**

Option take-up rates

The model may assume that all eligible policyholders will take up the option, and that the maximum additional benefit will always be taken.

You may see this approach (*ie* using the assumption that everyone exercises the option) described as the "conventional method" in past exam questions.

The assumption that everyone will exercise the option is obviously unlikely to be borne out in practice, but the method does have the advantage of simplicity.

If there are many possible dates on which an option may be exercised, or there is a choice from several alternative options, the model may assume that the worst option from the financial point of view of the company is chosen with probability one.

Alternatively, the model may use more sophisticated take-up rate assumptions which vary by exercise date or by alternative option. These would ideally be based on past experience.

You may see this approach (*ie* making an assumption about the proportion of policyholders that exercise the option) described as the "North American method" in past exam questions.

This method uses a more realistic assumption for the option take-up rate than the first approach suggested above. However, it may be difficult to obtain the data to make suitable assumptions (historically this data was only available in North America, hence the name).

Mortality rates

Typically, due to anti-selection, the expected mortality of lives who take up the option will be heavier than that of those who do not.

For example, the mortality of those who exercise the option may be assumed to be a higher percentage of the base mortality table.

For example, we may assume that those that exercise the option will experience mortality of 150% of the base mortality table.

Alternatively, an age loading may be applied (*eg* a policyholder of age x may be assumed to experience mortality of age $x+5$ years).

It may instead be assumed that the mortality experience of those who take up the option will be the Ultimate experience which corresponds to the Select experience that would have been used as a basis if underwriting had been completed as normal when the option was exercised. This would be consistent with an assumption that all eligible policyholders take up the option.

So this would be the approach to use for the "conventional method" described above.

As noted earlier, there should be a link between the assumed option take-up rates and the assumed mortality rates.

It may be assumed that the lives who do not take up the option will continue to experience the same level of mortality as would have been assumed without the existence of the option. However, this would mean that the average mortality for all lives has been assumed to be in excess of the base mortality assumption, since those taking up the option are assumed to experience higher mortality than this level.

For example, if we assume that lives that take up the option are "ultimate plus ten years" and the lives that do not take up the option are "ultimate", then the average of their mortality must be heavier than ultimate.

However, we would expect that the average mortality of all lives (both those that take and do not take the option) would be ultimate. To address this, the following alternative approach could be used.

An alternative assumption would therefore be that the mortality of those who do not take up the option is such that average mortality for all lives remains at the base expected level. The assumed mortality of those who do not take up the option would then be lower than this base level.

For example, under this alternative approach, we could assume that the lives that take up the option are "ultimate plus ten years" and that the average mortality will be ultimate. If the proportion that take up the option is 25% say, then we could determine a mortality assumption for the lives that do not take up the option, q', as follows:

$$q_x = 0.25q_{x+10} + 0.75q'$$

The assumed mortality of those who do not take up the option, q', will be lower than ultimate level. For example, we may find that $q' = q_{x-3}$.

Chapter 23 Summary

Investment guarantees

Examples of investment guarantees include:

- guaranteed minimum maturity values for both unit-linked and non-linked endowment contracts,
- guaranteed minimum surrender values for both unit-linked and non-linked contracts, and
- the ability to convert a lump sum into an annuity or vice versa on guaranteed terms.

An insurance company needs to be able to model the investment guarantee in order to quantify the extra liabilities that it will incur when the guaranteed amount exceeds the earned asset share. Liabilities can be assessed using:

- Option-pricing techniques: the value of the liabilities will be similar to the cost of a derivative which covers a similar guarantee or option to that which the company is offering.
- Stochastic simulation: the extra sums likely to be needed under the guarantee can be modelled stochastically by running a simulation of investment returns thousands of times.

Mortality options

Many life assurance contracts contain options whereby the policyholder can choose to extend the term or increase the level of cover at normal premium rates, *ie* without providing further medical evidence. To the extent that the option might be exercised by someone in poor health, the assurance company will bear a cost: the difference between the ordinary premium rate granted under the terms of the option and that which would have been granted had the life been underwritten.

Mortality options can be valued using cashflow projections. Assumptions are required for:

- option take-up rate
- benefit chosen
- mortality of those that exercise the option and those that do not
- expenses relating to the option.

This page has been left blank so that you can keep the chapter summaries together for revision purposes.

Chapter 23 Solutions

Solution 23.1

The company is at risk from low interest rates since the annuity may cost more to buy than the cash available.

The reverse is true if it invests to be able to pay the annuity. In this case, the risk is that interest rates are high and that the annuity the company has invested to be able to pay is worth less than the cash alternative.

Solution 23.2

The reason that the company cannot do this is that more or fewer policyholders may surrender in the future. Withdrawal rates are very variable particularly if interest rates and market values vary. For example, if interest rates rise and market values fall policyholders will see the opportunity to surrender and reinvest on attractive terms. In effect they are selecting against the company.

Solution 23.3

(i) ***Buying call options***

At the maturity (retirement) date of the policy, the insurance company will have cash available equal to the open market cash option. If interest rates are now lower than used in the guaranteed annuity rates, then the bonds that the company needs to buy to match the annuity payments will cost the company more money than the cash available from the policy maturity.

A call option essentially fixes a maximum purchase price for the bonds at the exercise date. So holding the option means that the company will not need to pay more money (than it can afford) for the bonds if interest rates do fall below the guaranteed level.

(This means that the current *market price* of the call option represents the expected cost of the annuity rate guarantee provided by the company.)

(ii) ***Buying interest rate swap options***

For this we would buy options that would permit the company to undertake "receive fixed/pay floating" swaps as from the retirement date, where the fixed rate in the swap agreement was equal to the interest rate in the guaranteed annuity basis. The option would only lead to the swap being undertaken if bond yields had fallen below the guaranteed rate at the retirement date.

Thereafter (assuming the option is exercised) the insurance company would have to keep its assets as cash, so that it could cover its obligation to "pay floating" under the swap agreement. Meanwhile the swap agreement will pay the insurer the income shortfall on its (cash) returns, making this up to the level of income required to cover the guaranteed annuity payments.

(So this time it is the current market price of the necessary swap options that represent the market valuation of the annuity rate guarantee.)

Solution 23.4

The company could purchase a bond put option.

Without the put option, the company is at risk if interest rates rise, its bonds fall in market value and the policy is surrendered for cash at retirement (or indeed at some earlier time). If it has a bond put option, it can sell those same bonds at a guaranteed price (or the put option will increase in price, which comes to the same thing).

Solution 23.5

The average is only an estimate because there may be random error (there are only a finite number of simulations), and there may also be errors in the structure of the model, and in the values of the parameters we have used.

Solution 23.6

The cost of the guarantee would be greater since the guarantee could bite at any time in the five-year period if the unit fund is less than 1,000 rolled up at 3½% *pa*.

The cost will be further increased by policyholders *choosing* to surrender more often when the cost of the guarantee is highest (this is called *financial selection*).

Solution 23.7

We will need to allow for the policyholder to exercise the guarantee the first time that it is in-the-money. Thus for each run we compare the unit fund of the policy at the end of the year with 1,000 rolled up at 3½% *pa*. The cost incurred for each run will be the difference between the guarantee and the unit fund on the first occasion that the guarantee is greater.

The average cost per run would then be calculated as for the maturity guarantee.

Solution 23.8

The guarantee for a without-profits policy is the basic sum assured. One approach to costing the guarantee would be to run the model with stochastic investment returns and, for a given level of premium, calculate the asset share at maturity allowing for expenses and the required profit deduction. The asset share for each run is compared with the sum assured and the loss calculated. The premium should then be varied until the probability and amount of loss are acceptable.

Solution 23.9

The policyholder cannot then immediately exercise the option on discovering that he is a poor risk, *eg* when he has just been diagnosed with a life-threatening disease.

Solution 23.10

The first is worst – if everyone with higher than select mortality takes up the option.

This means that the maximum possible total amount of gain has been made by the policyholders. This will *include* the cost to the company of insuring the few very high risk lives at standard premiums, *plus* the costs of insuring the other medium-high risks on the same terms.

Solution 23.11

The risk of mortality selection against the company is rather more severe if the policyholder can choose a term assurance contract rather than an endowment assurance under the option, for example. So the company might wish to restrict the availability of contracts such as term assurance, in these circumstances.

Solution 23.12

The existence of the mortality option increases the mortality risk. For example, an option to extend the term increases the time for which the insurer is exposed to risk. Similarly, an option to increase the sum assured increases the amount that is exposed to risk. The reserves need to be increased for the expected cost of the increased claims.

However, reserves will also need to increase to reflect the uncertainty in this expected cost. Alternatively, additional solvency capital will be required to cover these risks.

Before the option is exercised, the company is exposed to the risk that policyholders will selectively choose to exercise the option if their health is worse than for a select life.

Even after the option has been exercised, the company is still exposed to considerable uncertainty. As there is no further underwriting, it has no way of knowing the health of the lives that exercised the option. Reserves will need to be calculated on the assumption that a level of anti-selection has taken place.

Question 4.13

A life insurance company is currently reviewing its surrender values for regular premium without-profits endowment assurances.

(i) Describe the most important objectives of the company when determining surrender values. [5]

(ii) Suggest with reasons a suitable approach to calculating surrender values. [8]

(iii) Explain the problems caused by a sharp increase in interest rates in the context of surrendering policies. Describe how the approach recommended in part (ii) would cope with such problems. [5]

[Total 18]

Question 4.14

A life insurer sells level immediate annuities.

It has the choice of the following assets to invest in:

- a large issue of low grade, long-term, fixed-interest corporate bonds, issued by a large domestic multinational company
- a small issue of high grade, long-term, fixed-interest corporate bonds, issued by a small, private local company
- long-term, fixed-interest government bonds.

The corporate bonds provide a similar expected yield, which is significantly higher than the government bond yield.

(i) Discuss which bonds would be most appropriate to back the annuity liabilities. [6]

(ii) Describe how the choice of bonds would affect the discount rate used to value the liabilities in a market-consistent valuation. [4]

[Total 10]

Question 4.15

A life insurance company uses prospective methods and the following basis for calculating its surrender values and alteration terms:

- Mortality: AM92 (lives assumed to be Select at policy outset)
- Interest: 4% *pa*
- Expenses: Renewal: 2% of each premium
  Alteration or surrender: 120

A without-profits endowment assurance policy was issued exactly fifteen years ago to a life who was then aged exactly 40. The sum assured under the policy is 250,000, payable at the end of the year of death or at the end of the term of 25 years, and the annual premium is 6,200.

(i) Calculate the surrender value payable, if the policyholder surrenders the policy just before the premium now due. [2]

(ii) Instead of taking the surrender value, the policyholder asks to convert the policy into an endowment assurance maturing in five years. The sum assured is to remain the same. Calculate the revised premium. [2]

[Total 4]

If the insurer has chosen to hold the government bonds, it will not be able to include an illiquidity premium in its discount rate. [½]

If the insurer has chosen to hold either of the corporate bonds, then it may be able to include an illiquidity premium, … [½]

… if this is permitted by regulation / legislation. [½]

It is only generally appropriate to include an illiquidity premium for long-term, predictable liabilities … [½]

… which will be the case here if the insurer has a large enough portfolio to remove random fluctuations from the experience. [½]

The corporate bonds have similar yields overall, however, they are made up quite differently: [½]

- the multinational company's bonds have a high risk of default, but are very marketable / liquid, … [½]

- … so the majority of the yield margin (above the government bond yield) will reflect the default risk [½]

- the small private company's bonds have a lower risk of default, but are very unmarketable / illiquid, … [½]

  … so the majority of the yield margin (above the government bond yield) will reflect the illiquidity risk. [½]

Therefore, if the insurer held the multinational company's bonds, then the scope for including an illiquidity premium would be small, … [½]

… whereas if the insurer held the small private company's bonds, then a significant allowance may be made in the discount rate for illiquidity. [½]

[Maximum 4]

Solution 4.15

(i) ***Surrender value***

The surrender value is:

$$SV = \{\text{current prospective reserve}\} - \{\text{alteration expense}\}$$

$$= 250,000\ A_{55:\overline{10}|} - 0.98 \times 6,200\ \ddot{a}_{55:\overline{10}|} - 120$$

$$= 250,000 \times 0.68388 - 6,076 \times 8.219 - 120$$

$$= 120,911$$

[2]

(ii) ***Revision of premium***

Let P be the new annual premium, then:

$$120,911 = 250,000 A_{55:\overline{5}|} - 0.98 P\, \ddot{a}_{55:\overline{5}|}$$

[1]

So:

$$P = \frac{250,000 \times 0.82365 - 120,911}{0.98 \times 4.585} = 18,917$$

[1]
[Total 2]

Solution 4.16

(i) ***Why persistency rates are important in pricing***

If a life company pays a benefit upon surrender that is higher than asset share, the company will make a loss on that individual policy. The same will happen on policies that pay no surrender benefit when asset shares are negative. [1]

Similarly, paying a benefit which is less than asset share will give rise to a profit. [½]

Persistency rates are also very important for projecting future in-force volumes. For example, lower persistency rates would mean that less profit would be expected from the portfolio later in the policy term, as fewer policies would still be in force. [½]

Question 5.4

Describe the aim and main features of catastrophe reinsurance from the point of view of a ceding life insurance company. [5]

Question 5.5

Describe briefly:

(i) the reasons for underwriting life assurance proposals, and [4]

(ii) the effect that the standard of underwriting will have on the pricing of the contract. [2]

[Total 6]

Question 5.6

Describe the various ways in which a life insurance company can help manage its capital position using reinsurance. [7]

Question 5.7

A life office offers a without-profits endowment assurance policy. At the maturity date of the policy, a policyholder may effect a whole life without-profits policy at normal premium rates without medical evidence, for a fixed sum assured.

(i) Outline how cashflow projections could be used to calculate the extra premium to be charged for this mortality option. [3]

(ii) Describe how the cost of the option is impacted by the proportion of policyholders that exercise the option. [3]

[Total 6]

Question 5.8

A deferred annuity policy will pay a guaranteed amount of £20,000 annually in advance from age 60. Alternatively, at age 60, the policyholder may choose to take a lump sum payment of equivalent value, which must be used to purchase an annuity from another insurance company at that time.

The lump sum is guaranteed to be the higher of:

- the equivalent value calculated according to market rates of interest available at the time of conversion
- £250,000

The company expects interest rates to have the following probability distribution at the time of this policyholder's 60^{th} birthday:

- $P(i = 4\%) = 0.6$
- $P(i = 6\%) = 0.4$

Calculate the expected cost of this option to the insurance company as at age 60, assuming the company bases its annuity rates on AM92 ultimate mortality, and that policyholders always take the lump sum option when it is financially advantageous for them to do so. [4]

In return, the insurer will repay the loan to the reinsurer over a number of years. [½]

The repayments are contingent on the profits emerging from the business, and will not be paid if the profits do not materialise. [½]

The assets in the company's supervisory balance sheet will be increased by the amount of the loan. [½]

However, in some regulatory regimes the future loan repayments will not be included in the liability value shown in the balance sheet. [½]

The net effect is then to increase the value of the free assets in the company's balance sheet, so increasing capital. [½]

However, this form of financial reinsurance is not effective under accounting or supervisory regimes where credit can already be taken for the future profits and/or where a realistic liability has to be held in respect of the loan repayments. [1]

Risk premium reinsurance

Risk premium reinsurance can be used to reduce new business strain. [½]

The reinsurer provides a "loan" to the insurer in the form of reinsurance commission based on the volume of business reinsured. [½]

The loan repayments are spread over a number of years as additions to the reinsurance premiums. [½]

The reinsurer takes into account the expected lapse experience of the portfolio in determining the loan repayments. [½]

So, in a similar way to the contingent loan, the assets are increased by the reinsurance commission, but the liabilities may not need to increase to cover the additional reinsurance premiums. [½]

Again, this form of financial reinsurance is not effective under accounting or supervisory regimes where a realistic liability has to be held in respect of the loan repayments (*ie* the additional reinsurance premiums). [½]

Original terms reinsurance

Original terms reinsurance will reduce new business strain on individual policies as they are sold. [½]

A quota share basis will usually be used as this means that the benefits of the reinsurance will apply to all contracts (both big and small). [½]

The large initial commission will enhance the company's free assets, by capitalising the future expense loadings that the insurer is passing to the reinsurer through the reinsurance premiums. [1]

Further reduction in strain may be achieved if the reduction in net reserves exceeds the reinsurance premium, though the extent of this will depend on regulation. [½]

[Maximum 7]

Solution 5.7

(i) ***Method***

The cashflows arising from the whole life contract would be projected. [½]

These cashflows would include the sum assured payable on death … [½]

… and the premiums payable throughout the term of the whole life contract, … [½]

… to the extent to which the option is assumed to be taken up. [½]

The premium for the whole life contract would be based on the expected premium rates that would be charged to standard lives … [½]

… as at the maturity date of the endowment assurance. [½]

The projections should also allow for any expenses incurred, including the administration of the option. [½]

As the purpose of the valuation is pricing the option (rather than setting the liability) then allowance should also be made for the additional reserves that should be held, both before and after exercise. [½]

The cost of the option calculated above would then be respread over the premiums payable by all policyholders over the term of the endowment assurance policy. [½]

[Maximum 3]

(ii) ***Impact of the proportion that exercise the option on the cost of the option***

If a life, who is in good health and who would be expected to satisfy normal underwriting requirements, exercises the option, then the option will generate little or no additional costs. [½]

The exercise of the option by lives in poor health will generate considerable additional costs. [½]

So the total cost of the option depends on the health status of those who choose to exercise the option … [½]

… and the proportion of lives who choose to exercise the option. [½]

In general, the smaller the proportion who exercise the option, the worse will be the subsequent mortality experience of those exercising the option. [½]

If a substantial proportion exercise the option, then their subsequent mortality experience will on average be less extreme. [½]

The maximum possible cost occurs when every person with mortality worse than the standard rate takes up the option. This will *include* the cost to the company of insuring the few very high risk lives at standard premiums, *plus* the costs of insuring the other medium-high risks on the same terms. [1]

[Maximum 3]

Solution 5.8

If interest rates are 4%, then the normal lump sum would be:

$$20,000\ddot{a}_{60}^{@4\%} = 20,000 \times 14.134 = 282,680$$

As this is higher than the guaranteed amount, then there is no cost to the company of meeting the guarantee. [1]

If interest rates are 6%, then the normal lump sum would be:

$$20,000\ddot{a}_{60}^{@6\%} = 20,000 \times 11.891 = 237,820$$ [1]

The cost of the guarantee is then:

$$250,000 - 237,820 = 12,180$$ [1]

So the expected value of the cost of the guarantee is:

$$\sum_i E\left(C \mid i\right) \times P(i) = 0 \times 0.6 + 12{,}180 \times 0.4 = £4{,}872 \qquad [1]$$

[Total 4]

Solution 5.9

We have to find out the probability that, at age 60, the value of the assets will be less than the (expected present) value of the liabilities.

Define A as the random variable denoting the value of the assets at age 60. So the question tells us that:

$$A \sim N(250{,}000, (50{,}000)^2)$$

Define L as the random variable denoting the value of the liabilities at age 60. If there were **no** annuity option, then this value would always be £200,000.

This is because the company would calculate the annual amount of annuity as (for example) $\frac{200{,}000}{\ddot{a}_{60}^{(m)@\,i}}$, *which has an expected present value of*:

$$\frac{200{,}000}{\ddot{a}_{60}^{(m)@\,i}} \times \ddot{a}_{60}^{(m)@\,i} = 200{,}000$$

The actual expected cost of the annuity will be higher than this if the expected present value of an annuity of £15,000 *pa* (payable monthly in advance) and discounted at rate i is greater than £200,000, because in this case the guaranteed annuity would then be payable. This will happen if:

$$\ddot{a}_{60}^{(12)@\,i} > \frac{200{,}000}{15{,}000}$$

ie: $$\ddot{a}_{60}^{@\,i} > \frac{200{,}000}{15{,}000} + \frac{11}{24} = 13.7917$$

From the *Tables* we see that $\ddot{a}_{60}^{@4\%} = 14.134$ and $\ddot{a}_{60}^{@6\%} = 11.891$. Since discounting at a higher rate of interest reduces the present value, it follows that $\ddot{a}_{60}^{@8\%} < \ddot{a}_{60}^{@6\%}$. So the guarantee will only bite if $i = 4\%$.

Part 7 – Revision Questions

This part contains 100 marks of questions testing the material from the whole course. You may like to try these questions under exam conditions as a mock exam.

Question 7.1

A life insurance company sells only term assurances. It sells through both the insurance intermediary and own salesforce distribution channels.

(i) Explain how persistency can be a source of risk to this life insurance company. [4]

(ii) Discuss the differences in persistency experience for the two distribution channels. [2]

[Total 6]

Question 7.2

Describe the checks that an actuary could make on the policy data being used in the investigations of a life insurance company that sells a wide range of linked and non-linked products. [10]

Question 7.3

A small proprietary life insurance company issues only individual policies, all on a with-profits basis, and distributes profits using the revalorisation method. Only the savings profit is distributed to policyholders.

Describe the types of risk premium reinsurance that the company might have in place, with an explanation of why the reinsurance might be needed. Describe any other type of reinsurance that the company might need in addition to this, again giving reasons. [15]

Question 7.4

A life insurance company issues conventional without-profits term assurances and with-profits endowment assurances through insurance intermediaries. It sells similar volumes of both classes of business.

The company currently operates a uniform approach to underwriting all its business, seeking additional medical information from applicants of both policy types according to identical criteria of age, size of sum assured, health and lifestyle information obtained from the proposal form.

The company wishes to increase sales of both contracts, and you are considering whether to relax the company's underwriting criteria. Discuss this proposal. [7]

Question 7.5

A mutual life company writes all types of with-profits life and pension business. The surplus distribution method used for the with-profits business is the contribution method, with a small final dividend.

Describe the factors influencing the choice of assets comprising the fund dedicated to the company's with-profits contracts. [14]

Part 7 – Solutions

Solution 7.1

(i) ***Persistency risk***

In most investigations carried out by actuaries, assumptions will need to be made as to the future persistency / withdrawal rates under the contracts issued by the company. [½]

This could include full withdrawal (surrender or lapse) rates, partial (or income) withdrawal rates and paid-up rates, as applicable. [½]

Lapses are the most likely form of withdrawal for term assurances … [½]

… but other forms of withdrawal might be possible, *eg* surrenders for single premium policies or reductions in sum assured. [½]

There is therefore a "parameter" risk and there may also be a "model" risk. [½]

The parameters for other decrements (*eg* mortality) will be based on a model of the selective effect of withdrawals. Departures from the latter may invalidate the former. [½]

For products that pay a withdrawal benefit, whether there is a persistency risk depends on how the benefit compares with the policy asset share. For term assurance, there is normally no withdrawal benefit, so providing the asset share of the policy is positive (*ie* not too early on in the policy), a withdrawal would be a source of surplus. [1]

However, the company can be open to selection if healthy lives leave (for example if they find cheaper rates elsewhere), since the remaining lives may be of poorer health. [½]

The parameters for future expenses will also be based on a model of future withdrawals. Departures from the latter may invalidate the former. [½]

Such departures may also be a cause of the mismatch between the charges accruing to the company and its actual expenses. [½]

Random fluctuation risk may also arise, whereby the actual future withdrawal experience may not correspond with the model and parameters adopted, even though these adequately reflect the class of lives insured or to be insured. [½]

[Maximum 4]

(ii) ***Persistency experience***

Persistency experience might be best (*ie* withdrawals lowest) for the insurance intermediaries channel … [½]

… as the policyholder will have initiated the sale and so is more likely to take out a contract that meets his or her needs. [½]

Persistency experience might be low (*ie* withdrawals high) for the own salesforce channel if the salesperson pressurised the policyholder into buying a contract that he or she didn't really want. [½]

Persistency experience might also be better for the insurance intermediaries channel as their clients are generally financially sophisticated and will have received a high level of advice. [½]

However, term assurances are relatively simple contracts and so the level of financial sophistication and advice might have little impact on persistency … [½]

… unless the term assurances had a renewal or conversion option. [½]

Clients of insurance intermediaries also tend to have higher net worth and so will be more able to continue to pay the premiums. [½]

However, term assurances have relatively small premiums, so affordability may be less of an issue. [½]

[Maximum 2]

Solution 7.2

Data reconciliation checks

Where an investigation is being carried out on a regular basis, a reconciliation of the current data with those used for the previous one can be attempted. [½]

The data are first grouped in some sensible way, for example by year of entry within each broad contract type. [½]

Using data similarly grouped relating to business that has come onto the company's books and gone off between the dates of the two investigations, the following check is made for each group: [½]

$$\left\{\begin{array}{c}\text{data at}\\ \text{previous}\\ \text{investigation}\end{array}\right\} + \left\{\begin{array}{c}\text{business}\\ \text{come onto}\\ \text{the books}\end{array}\right\} - \left\{\begin{array}{c}\text{business}\\ \text{gone off}\\ \text{the books}\end{array}\right\} = \left\{\begin{array}{c}\text{data at}\\ \text{current}\\ \text{investigation}\end{array}\right\} \quad [1]$$

The above can check the following items of data for non-unitised business:

- the number of contracts
- the basic sum assured – or the equivalent benefit depending on the nature of the contract
- the office premium
- for with-profits contracts, the amount of any attaching bonuses

[½ each, maximum 1]

and for unitised business:

- the number of contracts
- the number of units actually allocated sub-divided by linked fund
- current premium payable
- current benefits available, *eg* amount of death cover, sickness benefit *etc*.

[½ each, maximum 1]

If relevant, the reconciliation also needs to allow for items such as:

- changes in the number of units allocated arising from switches between unitised funds [½]
- changes in the premium payable and benefits under existing contracts. [½]

(iii) ***Non-unit reserves***

The non-unit reserve is the present value of the excess of non-unit outgo over non-unit income. [½]

For this contract the non-unit outgo is the expenses plus any excess of the sum assured over the unit value payable on death. [½]

The non-unit income is the fund management charge. [½]

The expenses will increase with inflation over the term of the contract. [½]

The cost of the death benefit is likely to decrease over the term as the unit value will increase as further premiums are paid and (hopefully) as investments grow in value … [1]

… but the reducing cost of the death benefit will be offset slightly by the increasing rate of mortality as the policyholders grow older. [½]

The non-unit income is likely to increase over the term as it is a proportion of the unit value (which should grow over time as described above). [½]

So we would expect the non-unit cashflows to be negative in the early years (*ie* the outgo will exceed the income) … [½]

… but the non-unit cashflows should be positive in the later years … [½]

… becoming ever more positive as the assets grow. [½]

The size of the non-unit cashflows will depend on the following factors:

- a lower unit growth rate will make the cashflows less positive (or more negative) [½]
- a low premium will make fixed expenses relatively more important and so cashflows will be negative for longer [½]
- a low entry age will affect the cost of the death benefit as the sum assured will be higher but the probability of claiming will be lower. [1]

Prudential valuation

Under a regulatory regime which requires mathematical reserves to be prudent, the non-unit reserve is typically defined as the amount required to ensure that the company is able to pay claims and meet its continuing expenses without recourse to further finance. [½]

When calculating the reserve in the early years of the contract, the positive cashflows later in the term will be ignored, … [½]

… the calculation process will start with the last projection period in which the non-unit cashflows are negative … [½]

… and a positive non-unit reserve will be calculated so that all the future negative cashflows are "zeroised". [½]

The company should project forwards its non-unit cashflows on the reserving basis. [½]

The prudent nature of the reserving basis will make the expenses and death benefits larger and the fund management charges smaller … [½]

… and so will increase any positive non-unit reserves held. [½]

When calculating the reserves later in the term of the contract, once no more negative cashflows are expected, it may be permissible to hold a negative non-unit reserve. [½]

Regulations might require that the sum of the unit and non-unit reserve for a policy is not less than any guaranteed surrender value. [½]

If this is the case then negative non-unit reserves will not be allowed for this product as there is no surrender penalty (*ie* the full value of the units is paid on surrender). [½]

Regulations may also specify that a negative non-unit reserve can only be held if:

- the future profits arising on the policy with the negative non-unit reserve emerge in time to repay the "loan" [½]
- after taking account of the future non-unit reserves, there are no future negative cashflows for the policy, *ie* there should be no future valuation strain [½]
- in aggregate, the sum of all non-unit reserves is not negative. [½]

If a negative non-unit reserve cannot be held due to any of the above regulations, then the non-unit reserve will be set to zero. [½]

Best estimate valuation

Under a regulatory regime which requires mathematical reserves to be a best estimate valuation, the calculation would value all future non-unit cashflows (*ie* it would not disregard cashflows occurring after the last projection period in which there is a net outflow). [½]

The best estimate nature of the reserving basis will make the non-unit reserves more negative than for the prudent reserves described above. [½]

If the contract is written to be profitable, then the expected present value of the non-unit income should exceed the outgo. [1]

At the outset of the contract there will be a large non-unit outgo due to the initial expenses, but no income as 100% of the premium is allocated to units. [1]

So the expected present value of the income will be considerably greater than the outgo once the policy is in force. [½]

Hence a negative non-unit reserve could be held throughout the term of the contract. [½]

This reserve would become more negative throughout the term until the point at which the net income first became positive (as there would no longer be years of negative income offsetting the later years of positive income). [½]

Under a best estimate (or market-consistent) valuation, it would generally be the case that negative non-unit reserves can be held, so there would be no requirement to set these reserves to zero. [½]

[Maximum 10]

Question X5.1

A life company issues unit-linked personal pension contracts. These are written as pure endowments, with the proceeds at maturity being available either as cash, or to purchase a conventional without-profits immediate annuity.

The company guarantees that the annuity terms available at the specified retirement age will not be worse than those calculated on a 4.5% *pa* interest rate basis and according to a specified mortality basis.

(i) Describe how you would work out a prudent reserve to cover the cost of the guarantee using stochastic simulation. [5]

(ii) Describe how your answer to part (i) would differ if the policyholder could choose their retirement age at any time, rather than needing to specify it when taking out the policy. [1]

(iii) Describe how your answers to parts (i) and (ii) would differ if the company were costing the guarantee for pricing purposes rather than reserving purposes. [2]

As an alternative to valuing the guarantee using stochastic simulations, the company is considering valuing the guarantee using option pricing techniques. The value of the guarantee will then be set equal to the price of the derivative contracts that match the payments from the guarantee.

(iv) Outline two different derivative contracts that could be used to price this guaranteed annuity option. [2]

(v) Outline the problems that the company might face if it tried to hedge the guarantee by buying the derivatives in part (iv). [3]

[Total 13]

Question X5.2

An insurance company sells single premium term assurances with terms of one and two years. The sum assured for these contracts is 10,000 and is payable at the end of the year of death.

The company uses the following pricing basis for these standard contracts:

- Mortality: 1% *pa*
- Interest: 3.5% *pa* compound.

(i) Calculate the single premiums for the one year and two year contracts. [2]

The insurance company also sells special single premium term assurances with a term of three years. The initial sum assured for these contracts is 40,000. The sum assured is payable at the end of the year of death.

For these special contracts, the policyholder has the option to increase the sum assured to 50,000 without further evidence of health. The option can be exercised either at the end of the first year or at the end of the second year. If the option is exercised, the policyholder pays an additional single premium calculated using the standard terms as in part (i).

(ii) Discuss whether the special contract should be underwritten. [2]

The company uses the following pricing basis for these special contracts:

- Mortality after exercising the option: 3% *pa*
- Mortality otherwise: 1% *pa*
- Proportion of eligible lives who exercise the option at time 1: 50%
- Proportion of eligible lives who exercise the option at time 2: 30%
- Interest: 3.5% *pa* compound.

(iii) Calculate the single premium for the special contract using the basis above. [5]

The company has undertaken a review of its premium calculation methodology. One reviewer stated that the pricing basis only allows for mortality, option take-up rate and interest, but should also include other factors.

(iv) State the factors that should also be included in the premium calculation, and in each case state whether including the factor would increase or decrease the premium for the special term assurance. [4]

(v) Sales of the special term assurance have been low. Suggest possible reasons for the low sales. [4]

[Total 17]

(iv) ***Derivative contracts that match the guarantee***

A guaranteed annuity rate corresponds to a call option on the bonds that would be necessary to ensure the guarantee was met. [½]

The exercise price would be chosen so that the bonds generated the required return of 4.5%. [½]

Alternatively the guarantee can be mirrored by a swaption. [½]

This option would swap the floating rate returns at the option date for a fixed return of 4.5%. [½]

[Total 2]

(v) ***Problems with holding the derivative contracts***

Markers, please give credit for alternative examples of the problems described below.

The necessary derivatives may not be available in the market. [½]

For example, swaptions may not have a sufficiently long term to cover the life expectancy of the policyholders. [½]

The term of the annuity is unknown as it depends on the longevity of the policyholder. [½]

So the company will not know what term of bonds it should buy options for. [½]

The company does not know how large the unit-linked fund will be at retirement. [½]

So it does not know how many options / swaptions it should buy. [½]

Similarly, the company does not know how many policyholders will be eligible to exercise their option … [½]

… as some policyholders may die or surrender before retirement. [½]

The counterparty to the derivative contracts may default. [½]

[Maximum 3]

Solution X5.2

Comment

This topic is covered in Chapter 23.

(i) ***Calculation of standard premiums***

The premium for a one-year term assurance is:

$$P_1 = \frac{10,000 \times 0.01}{1.035} = 96.62 \qquad [1]$$

The premium for a two-year term assurance is:

$$P_2 = \frac{10,000 \times 0.01}{1.035} + \frac{10,000 \times 0.01 \times 0.99}{1.035^2} = 189.04 \qquad [1]$$

[Total 2]

(ii) ***Underwriting the special contract***

The terms of the contract are that the option can be exercised without *further* underwriting at that time. [½]

However, term assurances would normally be underwritten at *outset* (for the reasons described below). [½]

There is a very large sum at risk as the sum assured is considerably bigger than the premium. [½]

The sum at risk will increase further if the policyholder exercises the option. [½]

So the insurer can potentially make large losses if policyholders have heavier mortality than expected. [½]

Anti-selection may occur if applicants in poor health know that they will be able to obtain cover without underwriting. [½]

[Maximum 2]

(iii) ***Calculation of special premium***

Let P be the single premium for the special contract.

The expected present value of the cashflows in the first year is:

$$P - \frac{40,000 \times 0.01}{1.035} = P - 386.47 \qquad [1]$$

Half the policyholders do not exercise the option at the end of the first year. The other half exercise the option, pay the premium P_2 and then experience heavier mortality from that point. So, the expected present value of the cashflows in the second year is:

$$-0.5\left(\frac{40,000 \times 0.01 \times 0.99}{1.035^2}\right) + 0.5\left(\frac{P_2 \times 0.99}{1.035} - \frac{50,000 \times 0.03 \times 0.99}{1.035^2}\right)$$

$$= -184.84 + 90.41 - 693.13 \qquad [1½]$$

$$= -787.56$$

The 50% of policyholders that exercised the option at the end of the first year continue to experience heavier mortality. Of the remaining policyholders, 70% do not exercise the option at the end of the second year and experience standard mortality, and the other 30% exercise the option, pay the premium P_1 and then experience heavier mortality from that point. So, the expected present value of the cashflows in the third year is:

$$-0.5\left(\frac{50,000 \times 0.03 \times 0.99 \times 0.97}{1.035^3}\right) - 0.5 \times 0.7\left(\frac{40,000 \times 0.01 \times 0.99^2}{1.035^3}\right)$$

$$+0.5 \times 0.3\left(\frac{P_1 \times 0.99^2}{1.035^2} - \frac{50,000 \times 0.03 \times 0.99^2}{1.035^3}\right)$$

[2]

$$= -649.60 - 123.76 + 13.26 - 198.90$$

$$= -959.00$$

So the premium is given by

$$P = 386.47 + 787.56 + 959.00 = 2,133.03 \qquad [½]$$

[Total 5]

(iv) ***Factors to include in premium calculation***

Cashflow projections should be performed so that a full profit test of the premium can be made.

A profit loading should be included … [½]

… this would increase the premium. [½]

An allowance for initial, renewal and claim expenses should be included … [½]

… and the additional expenses associated with the option (*eg* notifying policyholders and processing their decisions) should also be included … [½]

… these would all increase the premium. [½]

An allowance for surrenders could be made … [½]

… this would reduce the premium if some lives in poor health surrendered … [½]

… but might increase the premium if a generous surrender value is paid. [½]

An allowance for reserves and solvency capital requirements should be included … [½]

… this would increase the premium … [½]

… as the providers of capital usually require a higher return than that earned on the reserves. [½]

[Maximum 4]

(v) ***Reasons for low sales***

The contract may not be a good match for customers' needs. [½]

There is no flexibility in the term of the contract and three years may be too short. [½]

There is no flexibility in the sum assured and customers may need more or less than the fixed amount. [½]

Competitors may be offering a similar product for a lower premium. [½]

Competitors may offer contracts with improved features, … [½]

… *eg* greater flexibility in the timing of exercising the option and/or the amount of extra sum assured that can be chosen … [½]

… or an option to convert the contract into an endowment assurance or whole life assurance. [½]

The public may be unaware of the product, *eg* due to a lack of advertising. [½]

The company may have a low reputation with the public, *eg* following a regulatory fine. [½]

The contract may be unpopular with intermediaries, *eg* if the company offers low commission. [½]

The economy may be in recession so that many potential customers cannot afford the contract. [½]

There may be little need for the contract if the state or employers offer generous benefits. [½]

[Maximum 4]

Solution X5.3

Comment

This topic is covered in Chapters 24 and 25.

The main form of reinsurance wanted will be risk premium reinsurance for contracts offering significant protection benefits. [1]

The main reason the company wants this sort of reinsurance is to reduce the impact of mortality fluctuations on free assets … [½]

