Subject SP5

Investment and Finance Principles

CMP Upgrade 2018/19

CMP Upgrade

This CMP Upgrade lists the main changes to the Core Reading and the ActEd core material since last year so that you can manually amend your 2018 study material to make it suitable for study for the 2019 exams. It includes replacement pages and additional pages where appropriate. Alternatively, you can buy a full replacement set of up-to-date Course Notes at a significantly reduced price if you have previously bought the full price Course Notes in this subject. Please see our 2019 *Student Brochure* for more details.

This CMP Upgrade contains information on changes made to:

- the Syllabus objectives
- the ActEd Course Notes and Series X Assignments.

1 Changes to the Syllabus objectives and Core Reading

The full syllabus for Subject SP5 exams in 2019 is available from the IFoA's website:
https://www.actuaries.org.uk/studying/curriculum-2019/investment-and-finance

There are quite a few changes to the instruction / command words. However, the topics covered are largely unchanged, with the main deletion of content being in section 5 (*eg* the removal of mergers & acquisitions).

In summary, the most significant changes to the core reading are:

- removal of some material under 'The Theory of Finance' section (*eg* mergers and acquisitions)
- addition of some additional material on behavioural finance
- extensive new material on portfolio theory
- removal of the Glossary.

2 Changes to the ActEd Course Notes

New chapter numbering

Chapters have been re-numbered, so that the first chapter is Chapter 1. So, what was Chapter 0 (Introduction to Subject ST5) is now Chapter 1 (Introduction to Subject SP5) and so on.

New Chapter (2019: Chapter 25)

A new chapter has been introduced to cover the small amount of new core reading on 'Solving problems' which appears in section 9 of the syllabus objectives. This full new chapter is supplied in an appendix to this CMP Upgrade document.

The Theory of finance (2018: Chapter 6 / 2019: Chapter 7)

As noted earlier, some objectives have been removed from section 5 of the syllabus. As a consequence, the first 3 sections that appeared in this chapter on 'The theory of finance' (*ie* the sections entitled 'Introduction to corporate finance', 'Agency theory', and 'Motives for mergers and acquisitions') have been removed.

What was section 4 on 'Behavioural finance' is now section 1. There is a new section 2 on 'The application of behavioural finance' and this appears in an appendix to this CMP Upgrade document. What was section 5 on 'Financial planning' is now section 3.

Risk control (2018: Chapter 17 / 2019: Chapter 18)

The first two sections that appeared in this chapter on 'Risk control' have been removed.

What was section 3 on 'Financial risks' is now section 1. What was section 4 'Important prerequisite material' has been removed. There is a new section 2 on 'Portfolio theory' and this appears in an appendix to this CMP Upgrade document.

Merging of the Q&A bank into the Course Notes

There is no Q&A bank for 2019. The majority of questions in the 2018 version of the Q&A bank have been merged into the Course Notes for 2019 – appearing at the end of each chapter within the Course Notes.

3 Changes to the X Assignments

The 2018 versions of the ST5 X assignments remain largely unchanged. However, question X2.1 (relating to mergers and acquisitions, which no longer appear in the syllabus) has been replaced with a new question on the application of behavioural finance.

IMPORTANT

Please note that we only accept the 2019 version of assignments for marking in the sessions leading to the 2019 exams. However, if you purchased the 2018 version of the SP5 X assignments and wish to submit your scripts for marking in 2019, please let us know at **ActEdMarkers@bpp.com**, and we will send, free of charge, an up-to-date version for you to attempt and submit.

4 Other tuition services

In addition to this CMP Upgrade you might find the following services helpful with your study.

Study material

We offer the following study material in Subject SP5:

- Mock Exam A
- Revision Notes
- Flashcards.

For further details on ActEd's study materials, please refer to our *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

Tutorials and Online Classroom

We currently offer the following in Subject SP5:

- a set of Regular Tutorials (lasting three full days), available either face-to-face or live online
- a Block Tutorial tutorials (lasting three full days)
- the Online Classroom – a great alternative to the tutorial (above), or as a valuable add-on to the tutorial.

For further details on the above, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **www.ActEd.co.uk**.

Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to our *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

5 Feedback on the study material

ActEd is always pleased to get feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course please send them by email to **SP5@bpp.com**.

2 The application of behavioural finance

2.1 Overview

Behavioural finance can have major implications for the way that market prices are formed both in a theoretical and practical sense and also has many potential applications in practice.

It is therefore important to have a broad grasp of the theoretical concepts underlying behavioural finance and how it differs from other areas of finance. What is usually termed 'modern financial economics' is based on the premise that participants (agents) in the markets are rational.

Question

List some attributes that would be consistent with 'rational' investing.

Solution

'Rational' investing may be characterised as:

- basing every decision on maximising wealth
- trading extra return with extra risk, where risk is volatility of future returns
- being fully informed on all potential investments
- being able to spread portfolios over a large number of investments without incurring excessive costs
- continuously reviewing all existing investments as well as reviewing potential new investments.

(*There are probably many more that you may have thought about*.)

Rationality

In technical terms rationality means that when they receive new information agents update their beliefs correctly in line with Bayes' law and that they act to maximise their Subjective Expected Utility over time (see, for example, Barberis and Thaler, 2003).

If agents are all rational we can conclude that the price of a security should equal its fundamental value so market prices can be taken as a measure of fair value. This is an enormously important result that is the basis for most financial theory particularly the efficient markets hypothesis. Most practitioners would perhaps doubt that every participant in the market is always rational but historically this was not considered a major theoretical difficulty as prices were considered to be set by informed agents who were rational.

This means that, so long as there are sufficient investors who manage large sums of money, and who do act rationally, it is not necessary in liquid markets to assume that every investor acts rationally. The weight of money managed by professional investors would be sufficient to take advantage of any perceived market anomaly. Market anomalies would therefore disappear.

Basically if prices deviated from the 'correct' value the most informed agents could make money from this and in the process move prices back to appropriate levels. So, for example, if irrational agents were overly pessimistic about the value of HSBC shares valuing them at 400p instead of a true value of 600p rational agents would make money by buying the shares until they reached the correct price of 600p.

Noise traders

A noise trader is someone that is heavily influenced by short-term noise (such as a news article, or another trader's opinion) and trade on that basis, ignoring other information and share fundamentals. They create volatility, and often generate a herd mentality which can push prices further and further from the theoretical 'rational' price.

Behavioural finance was given much theoretical support by work on noise trader risk and the limits to arbitrage which basically showed that it might be difficult and risky to take advantage of mis-pricing in the market (De Long et al, 1990). Even if an agent is sure that the market price of a security is incorrect it may be very risky to try to take advantage of this as irrational agents (known as noise traders) may make prices even more irrational in the short term.

The theory referred to above is built on the fact that, even if a rational investor has correctly assessed the price of a share, noise traders can push the price further in the wrong direction that investors would naturally think, making the rational investor's decision go horribly wrong in the short term. Without having the ability to cruise through the short-term losses and wait for the rational price to be achieved (a luxury that many institutional investors do not have) rational investors stay in the side lines.

Research on behavioural finance and its applications

Currently behavioural finance is a very active area of finance research. In a sense its theory and findings are necessarily more fragmented than modern financial economics in that there are so many possible departures from pure rationality. One might argue that market price can only equal true value in one way but it might depart from value in many ways and for many reasons.

Behavioural finance has many clear practical applications both in providing broad guidance about how markets function and also in potentially predicting specific areas of mispricing. It is interesting that some of the pioneers of behavioural finance are running or advising hedge funds trading on the basis of behavioural predictions.

The research on behavioural finance is very extensive so we only cover a selection of the most salient findings. We can classify the findings in a number of ways.

- **Some of the findings relate to observed market behaviour and these can be further subdivided into:**
  - **findings relating to overall, high level (macro) market behaviour**
  - **the cross-sectional behaviour of securities**
  - **findings based on quite specific anomalies.**
- **Other findings deal directly with the behaviour of individuals which will ultimately be reflected in market prices.**

Each of these potential classifications is explored further in the sections that follow.

2.2 Market behaviour at a macro level

If we firstly deal with broad issues of market behaviour at a very high level perhaps the most celebrated work is that by Nobel prize winner Robert Shiller. In a series of papers Shiller has argued that the volatility of stock prices is much greater than can be explained by rational models discounting the value of the dividends payable by the stocks (Shiller, 2003). This implies that stocks may be undervalued or overvalued relative to their price calculated by rational criteria.

Sentiment

Following an important paper by Barker and Wurgler there is now a great deal of interest in the role of sentiment in markets (Baker and Wurgler, 2006).

Investor sentiment is not an easy concept to define or measure precisely but has often been apparent in markets and recognised as important by practitioners.

A classic recent example of a period of very high sentiment would be the internet stock bubble (this occurred in the period 1996 to 2000, which although not that recent, is still relatively fresh in stock market investors' minds) **although market history seems to show recurring periods of high and low sentiment.**

Research shows that the performance of stocks with different characteristics depends considerably on sentiment. In particular, when sentiment is low one would expect high future returns on:

- **small stocks**
- **highly volatile stocks**
- **the stocks of unprofitable companies**
- **non-dividend paying stocks**
- **extremely high growth stocks**
- **distressed stocks.**

In contrast, when sentiment is high these patterns reverse or reduce.

A number of quantitative measures are used as proxies to measure sentiment including:

- **the average discount on closed-end funds**

  The 'discount' referred to here is the difference between the theoretical value of a trust as assessed by its net asset value per shares, and the actual market price.

- **share turnover**
- **the level of activity in the IPO market and the level of issue of new equity**
- **the level of the dividend premium, which is a proxy for the relative demand for dividend payers among investors.**

A composite sentiment index is compiled based on these measures and is available from the web-site of Professor Wurgler at NYU Stern School of Business.

Question

When sentiment is high, what would you expect to observe with respect to the discount on investment trusts, share turnover, IPO activity and the demand for high dividend paying shares?

Solution

We would expect the discount to narrow. Investors are positive on the future and searching for investments. If investment trusts stand at a discount, they may be purchased as investment opportunities, narrowing the discount to NAV.

As investors search for opportunities, buying and selling investments, we would expect turnover to rise.

If there is high demand from investors, those aiming to raise cash will take advantage of this fact, and issue new companies (IPOs) or shares in existing companies (rights issues) whenever possible.

High dividend paying shares are often called 'value' shares. Such shares generally do not fall very far in difficult times, and are therefore very popular when sentiment is poor. When sentiment is high, we would expect less demand for these shares are investors search for more risky, higher-growth opportunities for their cash.

By observing these features, researchers can assess whether the stock market was in positive or negative mood during particular periods in the past. They can then allocate certain observed behaviours as belonging to these market environments.

By doing this, it is observed that when sentiment is high, it has generally been good to invest in small shares, volatile shares, *etc*. Of course many items of research on such matters can be criticised as data mining.

Herding

Herding has received a lot of recent research attention (Bikhchandani and Sharma, 2000). In simple terms this is when investors make decisions based on what they observe others to be doing. Thus they buy or sell because others are buying or selling. This should be clearly distinguished from the situation where many investors do the same thing without reference to one another (spurious herding), for example, as a result of a major economic news announcement causing many investors to rationally sell equities.

It is commonly considered that as a result of the current low interest rate environment, many herding investors are borrowing to buy second or third residential properties as rental homes. This is 'spurious herding' because the investors are acting rationally based on the economic environment rather than blindly following the actions of others (although some are probably blindly following others!)

However, it still results in circumstances where investors are all facing the same way, and acting in the same way.

Herding is potentially bad for financial markets as it may increase volatility and possibly even threaten the integrity of the whole system.

There are a number of explanations for herding including:

- **the idea that investors may copy other investors that they perceive to be better informed about a particular situation**
- **the incentives facing fund managers (for example, the idea that if every manager performs equally badly you are unlikely to be sacked), or**
- **an intrinsic desire to conform.**

This is linked to 'regret aversion' that was mentioned earlier. If a decision to be different may involve a great deal of regret if it goes wrong, then the decision is avoided.

There is some evidence that investment analysts are prone to herding behaviour in their predictions. Herding is rather difficult to measure directly as it is hard to know an individual's true motives for an investment decision although a number of papers have developed quantitative measures to proxy it. Generally there is some evidence of herding amongst investment managers and this is more pronounced

- **in times of financial stress**
- **in less developed markets.**

2.3 The cross-sectional behaviour of stocks and other securities

Quite a number of studies have looked at the cross-section of stock returns. A classic paper by DeBondt and Thaler (1985) examined all stocks traded on the NYSE and ranked them by their past cumulative returns over the previous three years. The found that the average returns over the next three years of a portfolio formed from the worst performing stocks were almost 8% *pa* higher than those of a portfolio formed from the best performing stocks. They interpreted this as evidence of stock-market over-reaction.

This is the critical aspect that differentiates such 'cross-sectional-driven behaviours' from the 'high-level, macro-driven behaviours' discussed in the previous section. Cross-sectional-driven behaviours are those that can be linked to events in a previous time period.

A number of factors have been shown to be quite effective in explaining cross-sectional stock returns. Many of these factors have a long history of being used in the markets. It has been and continues to be strongly debated whether they result from behavioural factors or are in fact a rational response to other factors such as risk but certainly they are empirically very important. The size factor has been shown to be quite predictive with the stocks of small companies tending to outperform those of larger ones. Similarly 'value stocks' with high book to market values or low Price-to-Earnings ratios have been shown to outperform in numerous studies.

As mentioned earlier, small stocks tend to outperform in recovery situations when sentiment is high. Whereas value stocks with low PE ratios tend to outperform when sentiment is low.

Question

Define the 'book to price' ratio and state why value stocks have high book to price ratios.

Solution

The book to price ratio is defined as $\frac{\text{net asset value per share}}{\text{market price}}$ and is often less than one. Net asset value per share, or NAVps is defined as $\frac{\text{share cap and reserves} - \text{intangibles}}{\text{number of shares}}$.

Value stocks are often those with sound but unexciting fundamentals, where the market price drops as investors search for more exciting shares to invest in. As these shares have sound fundamentals, the accounts tend to show a high level of shareholder reserves, indicating that the company has plenty of non-current assets and not much debt. But if the share is unexciting, the market price will fall to low levels creating a high book to price ratio relative to other shares.

Cross-sectional momentum

An influential study by Jegadeesh and Titman (1993) shows that what is termed cross-sectional momentum is an important factor in predicting stock market returns. They group all stocks traded on the NYSE by return and show that the decile group that has performed worst over the last six months performs substantially worse than the group of the best performers over the next six months. Thus there is a cross-sectional momentum effect with relatively poor performing stocks continuing to perform poorly and well-performing stocks continuing to perform well.

Time series momentum

What is termed a time series momentum effect has also been well documented where the past performance of many financial instruments, including equity indexes, currencies, commodities, and sovereign bonds is considered in isolation rather than relative to other instruments. The past 12-month excess return of many instruments has been found to be a positive predictor of their future return over roughly the next year (Moskowitz et al, 2012).

Application of theory

The existence of behaviour that can be interpreted as over-reaction in the long run and under-reaction in the short run is clearly complex and a good deal of work has been done developing competing theoretical models that are well-grounded in both economic and psychological theory to explain this behaviour (Thaler, 1999).

So research has shown that:

- market over-reaction is clearly evident in markets, so investors should buy shares that have underperformed, waiting for the market recovery
- in the short term, momentum effects are evident in markets, so investors should buy shares that are outperforming as this outperformance will continue.

Clearly it is difficult to come to a clear theory that investors can use for portfolio decisions. Much depends on the market and economic environment as to which behaviours will dominate, and much will depend on what is considered ‘short-term’ and what is considered ‘long-term’.

A feature of the work on the cross-section of stock returns is that, whilst most academics and practitioners agree on the empirical facts regarding return patterns, the underlying reasons for the patterns are much more controversial and very difficult to totally pin down. Whilst many experts think behavioural explanations are the most plausible, others advocate rational causes such as higher returns being a reward for risk. The work by the academics Eugene Fama and Kenneth French certainly tends towards the latter view but they merit coverage even in a section devoted to behavioural finance for their ongoing and extremely influential work on quantifying, documenting and modelling factors affecting the cross-section of stock returns (see, for example, Fama and French, 1992). The web-site of Professor French at Dartmouth College contains extensive data on the historical performance of a wide range of factors.

2.4 Particular mood based anomalies

Numerous studies that have found market anomalies that seem best or at least plausibly explained by behavioural factors. Many of these sit rather uncomfortably with general theory but potentially could/should be taken account of in trading. In many cases the rationale is that the psychology literature shows that emotions and mood influence human decision making with good moods being associated with:

- **more positive evaluation of many sorts**
- **a tendency to make more positive choices**
- **using simple heuristics to make decisions.**

Some of the particularly striking examples amongst the numerous examples of such anomalies are mentioned below:

- **Calendar Effects – 'Blue Mondays' and Good Returns Pre-holidays. There is a vast literature on the link between dates and market returns. Interestingly, returns have often been found to be low on Mondays and high on the days before Public Holidays.**
- **Sunshine – Daily market returns in world markets have been found to be strongly correlated with the amount of morning sunshine in the city of the country's leading stock exchange (Hirshleifer and Shumway, 2003).**
- **Sports Results – A strong link has been established between national sports team losses and declines in the stock market of that country (Edmans et al, 2007).**
- **Aviation disasters have been shown to lead to losses of stock market value which are a large multiple of the estimated actual economic losses. A finding presumably due to induced negative sentiment (Kaplanski and Levy, 2010).**

2.5 The behaviour of individuals

Studies have been conducted on the way individuals can be expected to behave based on theory from psychology and also on the way they actually do behave when their actions can be observed. These findings have clear implications for the markets.

Numerous findings show that individuals do not act as fully rational economic agents attempting to maximise their utility. One of the most celebrated departures from conventional rationality is described by Prospect theory which was developed by Daniel Kahneman and Amos Tversky (1979).

Question

Outline 'prospect theory' in general terms.

Solution

Prospect theory indicates that:

- when faced with potential gains, investors are generally risk-averse
- when faced with potential losses, investors can become risk-seeking.

This theory has many important implications and has been very extensively employed in behavioural finance. In the simplest terms the theory models the fact that people underweight outcomes that are probable rather than certain. This implies that value is assigned to gains and losses rather than final wealth and probabilities are replaced by more appropriate decision weights.

The study of prospect theory has uncovered a number of features. The most commonly understood is that a person's utility function for gains tends to be risk-averse, and the utility function for losses can be risk-seeking. However, there are some other outcomes, including the fact that the utility function for losses is much steeper than the utility function for gains. In other words people detest losses, but only modestly like gains from their current wealth situation.

Another outcome is that small probability events are generally over-estimated, such that an event that has a 1% chance of happening may be perceived in the mind as having a 2% or even a 5% chance of happening. However, when probabilities rise towards the certain, the weights applied by people are less than they are in reality.

The theory is an important one for explaining the success of insurance, where a person is expected to part with a small premium (say £10) to insure against an unlikely event (say a 1% chance of losing £500). In mathematical terms, the insurance does not seem to make sense. But if the person making the decision applies a 5% chance to the event occurring, then the insurance does make financial sense from a utility perspective. However, ask someone to pay £85 to insure against a 90% chance of losing £100, and you are unlikely to sell many products.

Considerable work has been done on the way that individuals form expectations. Many of the factors are summarised in a survey paper by Barberis and Thaler (2003):

- overconfidence
- optimism
- representative bias
- belief preservation
- anchoring
- availability bias.

Overconfidence

People tend to be overconfident when making estimates both regarding the confidence intervals around their estimates and the probability of particular events occurring.

So, for example, an individual estimating a range for the £/$ in a year's time will tend to make the range far too small, based on the mathematical volatility of the currency rate in the past. Likewise, someone estimating the chance of a change of government in the coming year will tend to make the probability much higher than it perhaps is.

Question

Outline the two linked themes that are observed, and are caused by overconfidence.

Solution

One linked theme is called 'confirmation bias' where individuals seek out information that confirms their own personal opinion. This boosts their feeling of confidence and certainty that they are right.

A second linked theme is 'hindsight bias' whereby individuals feel after an event that the predicted it successfully. Often evidence suggests that they perhaps did not predict the event prior to it occurring, but the feeling of overconfidence is boosted by the feeling that the individual was correct and accurate in the past.

Optimism

People tend to overestimate their own abilities.

This one needs no explanation!

Representative Bias

People are poor at making good inferences about probabilities. They often put too much emphasis on particular features of the sample as opposed to the likely features of the whole population.

For example, if you know of three people that had their car damaged while parked in a railway car park, then you may be tempted to believe that there is a very high chance that your own car will be damaged if you park it in a railway car park. Realistically the sample of three is far too low to make this decision. Similar behaviour affects financial decisions. It is similar to 'availability bias' mentioned earlier.

Belief Preservation

Once people have formed a belief they tend to be overly reluctant to change it even in the face of strong contrary evidence.

It is commonly observed that when an investor buys a share that generates a substantial loss, they are unwilling to sell out and cut their losses. They prefer to hang on and hope that the share recovers, to help restore their self-belief, even if this means taking the risk that the share will continue to fall and generate further losses.

Anchoring

When forming estimates people often tend to start from an initial value which may be more or less arbitrary and then make adjustments away from that value. Experimental evidence shows that the adjustments tend to be too small so people are said to be too anchored on the initial value.

This behavioural theme was mentioned earlier in the chapter. Suppose an investor is deciding whether to sell an underperforming share which has fallen due to some event that was unforeseen at the time the investor purchased the share. The anchor might be the price at which the shares was purchased. The adjustment based on the new information will generally be less than it perhaps should be, and the investor will end up with the conclusion that the share is worth more than its current price.

Availability bias

When estimating probabilities people tend to focus overly much on more recent and more salient events.

After a market crash, investors estimates of the chance of a market crash in the near future increase. They are influenced by the recent crash that is easily 'available' to them in their minds.

2.6 Direct investigation of individuals' trades and portfolios

A body of research has investigated the trades and portfolios of stock market participants to find direct evidence of their behaviour. Brad Barber and Terrance Odean were pioneers in this area and produced some seminal papers by investigating the trades made by the customers of a large brokerage house. They found that individuals trade too much, falsely believing they can pick winners, whereas they are actually losing money because of trading costs and poor trades (Barber and Odean, 2000). Clearly this activity is consistent with overconfidence.

In a later study they find that men trade significantly more than women which is consistent with research in psychology showing that men tend to be more overconfident (Barber and Odean, 2001).

Odean has also shown that individuals are prone to irrationally realise the profits of winning as opposed to losing trades, a finding known as the disposition effect which is closely related to Prospect theory (Odean, 1998).

So when an investor has made an unrealised gain, there is a tendency to realise it and bank the cash. Whereas when an investor has made an unrealised loss, there is a tendency to hold on to the investment, and 'gamble' it in order to reduce the loss. This is similar to prospects theory, which says that investors are generally risk-averse with their gains, and risk-seeking with losses. Many behavioural themes overlap, making it a difficult area to develop useable theorems, or even to classify observations. It is not for the more scientific of actuarial students.

Work on the portfolios of investors has shown that individuals tend to hold portfolios that are far from the optimum and may use inefficient means of diversification. US investors hold under-diversified portfolios and this problem is worse for younger, low-income, less-educated and less-sophisticated investors. Furthermore this lack of diversification seems consistent with a number of behavioural biases (Goetzmann and Kumar, 2008). There is evidence from US pension plan data that individual investors use very naive methods of investing including often simply using a '1/N heuristic' where they simply divide their funds equally between the N funds available to them (Benartzi and Thaler, 2001).

Question

State a possibly reason why individual investors invest their money is (say) 30 equal amounts rather than a more sophisticated approach.

Solution

There are a number of possible reasons for this, including the following:

- Individual investors do not have access to the research that institutional investors have. They trust that institutional investors keep the market relatively efficient. Of the two decisions to be made (which 30 shares to select, and how much to invest in each) the first of these is considered to be more important than the second, therefore equal amounts are invested.
- Individual investors who don't trade very much and tend to 'buy and hold' have a habit of waiting until their savings cash, or pension cash, reaches a certain amount (say £10k) and then investing it. This will result in a fixed number of investments of approximately the same amount.
- Individual investors invest more heavily in large cap shares. The size of these companies is extremely large in all cases, and the investor's holdings in each share is almost infinitesimally small. Tapering the proportion of each investment may seem pointless.

2.7 Discussion and conclusions

The literature on behavioural finance is very extensive and the evidence supporting many of its findings is much too strong to ignore. This means it is important that practitioners are not ignorant of this work and its potential implications.

This section has briefly introduced some of the most important findings from the area but is far from comprehensive. Indeed one of the problems with behavioural finance is that it is difficult to distil the findings down to a small number of general theories.

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Chapter 7 Summary

Behavioural finance

The field of behavioural finance relates to the psychology that underlies and drives financial decision-making behaviour. Eight common themes include:

1. anchoring and adjustment
2. prospect theory
3. framing (and question wording)
4. myopic loss aversion
5. estimating probabilities
6. overconfidence
7. mental accounting
8. effect of options.

In addition, research has shown evidence of:

- herding
- cross-sectional momentum (*ie* good returns in previous time periods affect expected returns in the next time period)
- the influence of mood in investment performance (sunshine, sports results, ...)
- belief preservation.

Financial planning

Long-term financial planning (capital budgeting) commonly looks 3 to 5 years ahead and assesses the capital required for long-term projects. It involves the development of *business plans* setting out the firm's anticipated product development and sales objectives. These are then converted into *financial plans*, which convert the business plans into future cashflows. *Sensitivity analysis* should be used when developing the plans.

Short-term financial planning (cash management) often takes the form of a 12-month 'rolling' plan and revolves around the analysis of working capital requirements. It involves the consideration of:

- trade credit management (trade receivables and payables, *ie* debtors and creditors)
- cash management
- stock and inventory policy (raw materials and finished and unfinished goods)
- non-cash elements in the projected accounts.

2 Portfolio theory

2.1 Introduction

***Mean-variance portfolio theory*, sometimes called *modern portfolio theory* (MPT), specifies a method for an investor to construct a portfolio that gives the maximum return for a specified risk, or the minimum risk for a specified return.**

If the investor's utility function is known, then MPT allows the investor to choose the portfolio that has the optimal balance between return and risk, as measured by the variance of return, and consequently maximises the investor's expected utility.

However, the theory relies on some strong and limiting assumptions about the properties of portfolios that are important to investors. In the form described here the theory ignores the investor's liabilities although it is possible to extend the analysis to include them.

By ignoring actuarial risk – the risk that the investor fails to meet his or her liabilities – the theory as presented here is severely limited. In its defence, MPT was the first real attempt to use statistical techniques to show the benefit of diversification for investors.

The application of the mean-variance framework to portfolio selection falls conceptually into two parts:

1. **First the definition of the properties of the portfolios available to the investor – the *opportunity set*.**

   Here we are looking at the risk and return of the possible portfolios available to the investor.

2. **Second, the determination of how the investor chooses one out of all the *feasible portfolios* in the opportunity set.**

   This is the determination of the investor's *optimal* portfolio from those available.

2.2 Assumptions of mean-variance portfolio theory

The application of mean-variance portfolio theory is based on some important assumptions:

- all expected returns, variances and covariances of pairs of assets are known
- investors make their decisions purely on the basis of expected return and variance
- investors are non-satiated
- investors are risk-averse
- there is a fixed single-step time period
- there are no taxes or transaction costs
- assets may be held in any amounts, *ie* short-selling is possible, we can have infinitely divisible holdings, and there are no maximum investment limits.

These assumptions are all discussed below.

2.3 Specification of the opportunity set

In specifying the opportunity set it is necessary to make some assumptions about how investors make decisions. Then the properties of portfolios can be specified in terms of relevant characteristics. It is assumed that investors select their portfolios on the basis of:

- **the expected return**
- **the variance of that return**

over a single time horizon. Thus all the relevant properties of a portfolio can be specified with just two numbers – the mean return and the variance of the return.

The variance (or standard deviation) is known as the *risk of the portfolio*.

So, according to MPT, variance of return and expected return are all that matter – this is a key assumption. Other key factors that might influence the investment decision in practice are ignored. These include:

- the suitability of the asset(s) for an investor's liabilities
- the marketability of the asset(s)
- higher moments of the distribution of returns such as skewness and kurtosis
- taxes and investment expenses
- restrictions imposed by legislation
- restrictions imposed by the fund's trustees.

Finally, we should note that the length of the time horizon, which is likely to vary between investors, is not specified.

To calculate the mean and variance of return for a portfolio it is necessary to know the expected return on each individual security and also the variance/covariance matrix for the available universe of securities.

The variance/covariance matrix shows the covariance between each pair of the variables. So, if there are three variables, 1, 2 and 3 say, then the matrix has the form:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

where c_{ij} is the covariance between variables *i* and *j*.

It follows that:

- $c_{ij} = c_{ji}$ and so the matrix is symmetric about the leading diagonal
- c_{ii} is the variance of variable *i*.

Note that this means that with *N* different securities an investor must specify:

- *N* expected returns
- *N* variances of return
- $N(N-1)/2$ covariances.

Whilst estimates of the required parameters can be obtained using historical data, these are unlikely to prove reliable predictors of the future behaviour of investment returns and it may be necessary to adjust the historical estimates in the light of the other factors.

Question

If you assume that there are 350 shares in an equity index (as there are in the FTSE 350), how many items of data need to be specified for an investor to apply MPT?

Solution

The required number of items of data is:

$$350+350+\frac{350\times 349}{2}=61,775$$

Note that this ignores all the other available investments that are not included in the FTSE 350 Index, *eg* FTSE Smallcap, FTSE Fledgling, unquoted equities, non-UK equities, property, bonds *etc*.

This requirement for an investor to make thousands of estimates of covariances is potentially a major limitation of mean-variance portfolio theory in its general form.

2.4 Efficient portfolios

Two further assumptions about investor behaviour allow the definition of *efficient portfolios*.

The assumptions are:

- ***Investors are never satiated*. At a given level of risk, they will always prefer a portfolio with a higher return to one with a lower return.**
- ***Investors dislike risk*. For a given level of return they will always prefer a portfolio with lower variance to one with higher variance.**

This means that investors prefer 'more to less' and that they are risk-averse.

A portfolio is *inefficient* if the investor can find another portfolio with the same (or higher) expected return and lower variance, or the same (or lower) variance and higher expected return.

A portfolio is *efficient* if the investor cannot find a better one in the sense that it has both a higher expected return and a lower variance.

So, an *efficient* portfolio is one that isn't *inefficient*.

Thus, every portfolio – including those that consist of a single asset – is either efficient or inefficient.

Once the set of efficient portfolios has been identified all others can be ignored.

This is because an investor who is risk-averse and prefers more to less will never choose an inefficient portfolio. The set of efficient portfolios is known as the *efficient frontier*.

However, an investor may be able to rank efficient portfolios by using a utility function, as shown in Section 2.5 below

Recall from earlier subjects that if we know an individual's utility function then we can describe their attitude towards risk and return. If the assumption that investors make their decisions purely on the basis of expected return and variance holds, then this attitude towards risk and return can equally be described by indifference curves. Indifference curves join points of equal expected utility in expected return-standard deviation space, *ie* portfolios that an individual is indifferent between.

Suppose an investor can invest in any of the N securities, $i = 1,...N$. A proportion x_i is invested in security S_i.

Note that:

- x_i is a proportion of the total sum to be invested
- given infinite divisibility, x_i can assume any value along the real line, subject to the restriction that $\sum x_i = 1$
- we have not specified the nature of the N securities.

The return on the portfolio R_P is:

$$R_P = \sum_i x_i R_i$$

where R_i is the return on security S_i.

The expected return on the portfolio is:

$$E = E\left[R_P\right] = \sum_i x_i E_i$$

where E_i is the expected return on security S_i.

The variance is:

$$V = \text{var}\left[R_P\right] = \sum_j \sum_i x_i x_j C_{ij}$$

where C_{ij} is the covariance of the returns on securities S_i and S_j and we write $C_{ii} = V_i$.

So, the lower the covariance between security returns, the lower the overall variance of the portfolio. This means that the variance of a portfolio can be reduced, by investing in securities whose returns are uncorrelated, *ie* by diversification.

Question

How can the covariance of the returns on securities S_i and S_j be expressed in terms of the correlations of the returns on securities S_i and S_j?

Solution

The covariance C_{ij} is equal to $\rho_{ij}\sigma_i\sigma_j$

where:

σ_i = standard deviation of security *i* returns

σ_j = standard deviation of security *j* returns

ρ_{ij} = correlation coefficient between security *i* returns and security *j* returns

2.5 The case of two securities

We can derive expressions for the mean and the variance of portfolio returns in simple cases.

If there are just two securities, S_A and S_B, the above expressions reduce to:

$$E = x_A E_A + x_B E_B$$

and $$V = x_A^2 V_A + x_B^2 V_B + 2x_A x_B C_{AB}$$

As the proportion invested in S_A is varied a curve is traced in $E-V$ space.

The minimum variance can easily be shown to occur when:

$$x_A = \frac{V_B - C_{AB}}{V_A - 2C_{AB} + V_B}.$$

The variance of the portfolio return is:

$$V_P = x_A^2 V_A + x_B^2 V_B + 2x_A x_B C_{AB}$$

$$= x_A^2 V_A + (1-x_A)^2 V_B + 2x_A(1-x_A)C_{AB}$$

We want to choose the value for x_A that minimises the variance V_P. To do this, we differentiate and set to zero:

$$\frac{dV_P}{dx_A} = 2x_A V_A + 2(1-x_A)V_B(-1) + 2(1-x_A)C_{AB} - 2x_A C_{AB} = 0$$

$$\Leftrightarrow \quad 2x_A V_A - 2V_B + 2x_A V_B + 2C_{AB} - 4x_A C_{AB} = 0$$

$$\Leftrightarrow \quad x_A(V_A + V_B - 2C_{AB}) = V_B - C_{AB}$$

$$\Leftrightarrow \qquad x_A = \frac{V_B - C_{AB}}{V_A + V_B - 2C_{AB}}$$

As an example consider the case where:

$$E_A = 4\% \qquad V_A = 4\%\% \qquad (\sigma_A = 2\%)$$

$$E_B = 8\% \qquad V_B = 36\%\% \qquad (\sigma_B = 6\%)$$

We now let the correlation coefficient between the two securities vary by considering ρ_{AB} equal to –0.75, 0, and +0.75 in turn. The results are plotted in Figure 18.1 where the vertical axis represents expected values of return and the horizontal axis represents standard deviation of return. In this space $(E-\sigma)$ the curves representing possible portfolios of the two securities are hyperbolic. It is possible to plot the same results in $E-V$ space, where the lines would be parabolic.

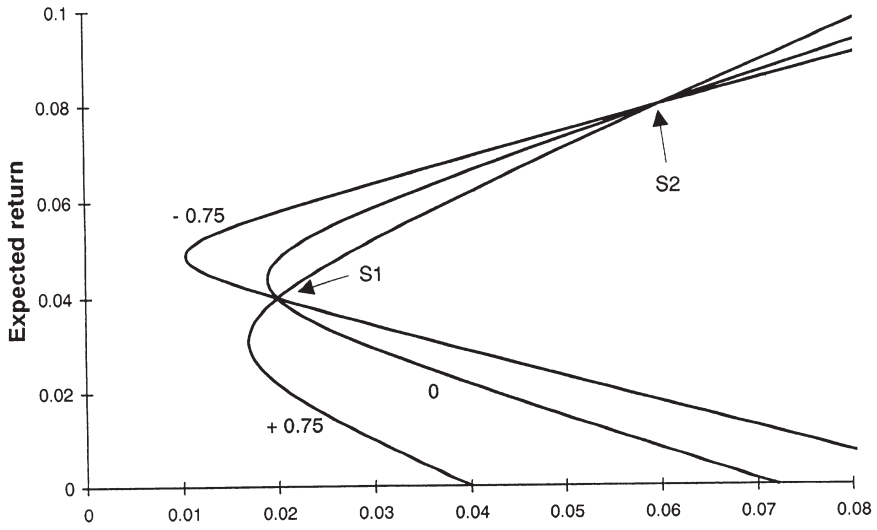

Figure 18.1

Note that S_A and S_B are denoted by S1 and S2 respectively in Figures 18.1 and 18.2, *ie* the points S1 and S2 represent the expected return and standard deviation for portfolios consisting entirely of S_A and S_B respectively.

Question

What are the co-ordinates of the point of minimum variance in Figure 18.1 in the case when $\rho_{AB} = 0$? Comment on this result.

Solution

The minimum variance occurs when:

$$x_A = \frac{V_B - C_{AB}}{V_A + V_B - 2C_{AB}}$$

If $\rho = 0$, then this reduces to $x_A = \frac{V_B}{V_A + V_B}$, so that in this instance:

$$x_A = \frac{0.0036}{0.0004 + 0.0036} = 0.9 \text{ and so } x_B = 0.1.$$

Thus, the resulting portfolio has expected return and variance given by:

$$E_P = x_A E_A + x_B E_B$$
$$= 0.9 \times 0.04 + 0.1 \times 0.08$$
$$= 0.044$$

and: $\sigma_P^2 = x_A^2 V_A + x_B^2 V_B + 2 x_A x_B C_{AB}$

$$= 0.81 \times 0.0004 + 0.01 \times 0.0036 + 2 \times 0.9 \times 0.1 \times 0$$
$$= 0.01897^2$$

ie $\sigma_P = 0.019$ or 1.9%

This is less than both of the individual security standard deviations of 2% and 6%, illustrating the benefits of diversification.

Figure 18.2 shows combinations of securities with correlation coefficients of +1, 0 and –1. For coefficients +1 and –1, it is possible to obtain risk-free portfolios, with zero standard deviation of return,

Note that in each of the three cases, the set of efficient portfolios consists only of those portfolios above the point of minimum variance.

If ρ is equal to +1 or –1, then there exists a *risk-free* portfolio with $V = 0$:

- if $\rho = -1$, it involves positive holdings of both securities, *ie* $x_1, x_2 > 0$
- if ρ = +1, it involves a negative holding of S_B (S2) and a positive holding of S_A (S1).

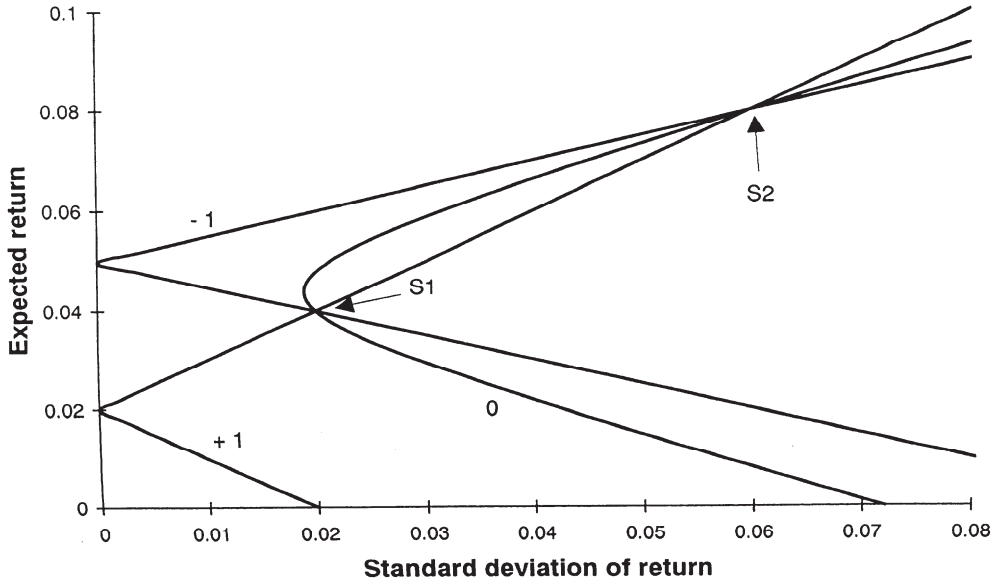

Figure 18.2

2.6 N-security case

When there are *N* securities the aim is to choose x_i to minimise *V* subject to the constraints:

$$\sum_i x_i = 1$$

and $E = E_P$**, say,**

in order to plot the minimum variance curve.

The aim is to choose the proportions to invest in each possible security in a way that minimises risk subject to the constraints that:

- all the investor's money is invested somewhere and
- the expected return on the portfolio is set equal to the desired level.

An alternative approach would be to *maximise E* subject to:

$$\sum_i x_i = 1 \text{ and } V = V_P \text{ say.}$$

However, the first approach is usually easier.

Note carefully that *E* and *V* without the subscripts are the *portfolio* expected return or variance, *ie* the quantities that we are optimising, and that E_P and V_P are the specified values used in the *constraints*.

Lagrangian multipliers

One way of solving such a minimisation problem is the method of Lagrangian multipliers.

The Lagrangian function is:

$$W = V - \lambda(E - E_P) - \mu\left(\sum_i x_i - 1\right)$$

or $$W = \sum_i \sum_j C_{ij} x_i x_j - \lambda\left(\sum_i E_i x_i - E_P\right) - \mu\left(\sum_i x_i - 1\right)$$

where:

- V, E and x_i are defined as above
- E_P and 1 are the constraining constants and
- λ and μ are known as the *Lagrangian multipliers*.

Remember that we are trying to *minimise* the variance V, subject to the expected return and 'all money invested' constraints.

Question

Recall the example in the Core Reading involving Securities S_A and S_B illustrated in Figure 18.1, in which:

$$E_A = 4\%, \quad V_A = 4\%\%$$

$$E_B = 8\%, \quad V_B = 36\%\%$$

Write down the Lagrangian function W in the case where the correlation coefficient is $\rho_{AB} = 0.75$

Solution

Here the Lagrangian function is:

$$W = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_A x_B \rho_{AB}\sigma_A\sigma_B - \lambda(E_A x_A + E_B x_B - E_P) - \mu(x_A + x_B - 1)$$

$$W = 4x_A^2 + 36x_B^2 + 2x_A x_B \times 0.75 \times 2 \times 6 - \lambda(4x_A + 8x_B - E_P) - \mu(x_A + x_B - 1)$$

ie $$W = 4x_A^2 + 36x_B^2 + 18x_A x_B - \lambda(4x_A + 8x_B - E_P) - \mu(x_A + x_B - 1)$$

To find the minimum we set the partial derivatives of W with respect to all the x_i and λ and μ equal to zero. The result is a set of linear equations that can be solved.

The partial derivative of W with respect to x_i is:

$$\frac{\partial W}{\partial x_i} = 2\sum_j C_{ij} x_j - \lambda E_i - \mu$$

The partial derivative of W with respect to λ is:

$$\frac{\partial W}{\partial \lambda} = -\left(\sum_i E_i x_i - E_P\right)$$

and with respect to μ is:

$$\frac{\partial W}{\partial \mu} = -\left(\sum_i x_i - 1\right)$$

Setting each of these to zero gives:

$$2\sum_j C_{ij} x_j - \lambda E_i - \mu = 0 \text{ (one equation for each of } N \text{ securities)}$$

$$\sum_i x_i E_i = E_P$$

$$\sum_i x_i = 1$$

These $N+2$ equations in $N+2$ unknowns (first-order conditions) can be solved to find the optimal values of the security proportions, *ie* the x_i's, as functions of the portfolio expectation E_P. These functions can then be substituted into the expression for the portfolio variance, the resulting expression for the portfolio variance as a function of the portfolio expectation being the equation of the minimum variance curve. It is the top half of this curve, *ie* above the point of global minimum variance, that is the efficient frontier.

Question

Write down the above conditions for the example in the preceding question, where we had:

$E_A = 4\%, \quad V_A = 4\%\%$

$E_B = 8\%, \quad V_B = 36\%\%$

and $\rho_{AB} = 0.75$.

Solution

Differentiating the Lagrangian function given in the solution to the preceding question gives the first-order conditions:

$$\frac{\partial W}{\partial x_A} = 8x_A + 18x_B - 4\lambda - \mu = 0$$

$$\frac{\partial W}{\partial x_B} = 18x_A + 72x_B - 8\lambda - \mu = 0$$

$$\frac{\partial W}{\partial \lambda} = -\left(4x_A + 8x_B - E_P\right) = 0$$

$$\frac{\partial W}{\partial \mu} = -\left(x_A + x_B - 1\right) = 0$$

We now generalise to any *E* and *V* …

… rather than the specific values of E_P and V_P .

The solution to the problem shows that the minimum variance *V* is a quadratic in *E* and each x_i is linear in *E*.

The usual way of representing the results of the above calculations is by plotting the minimum standard deviation for each value of E_P as a curve in expected return–standard deviation $(E-\sigma)$ space.

Recall that this is done for the two-security case in Figures 18.1 and 18.2 earlier in this chapter. As *V* is quadratic in *E*, so the resulting minimum standard deviation curve is a hyperbola in $E-\sigma$ space and a parabola in $E-V$ space. Hereafter we always consider $E-\sigma$ space unless stated otherwise.

In this space, with expected return on the vertical axis, the *efficient frontier* is the part of the curve lying above the point of the global minimum of standard deviation.

All other possible portfolios are inefficient.

Remember, investors make their decisions purely on the basis of expected return and variance. This means that, if the returns on securities are normally distributed, the returns can be characterised by just the mean and the variance. In this case MPT could be used to select optimal portfolios.

In fact, it can be shown that normality of returns is not a necessary condition for the selection of optimal portfolios. There is a more general class of distributions called the elliptically symmetrical family, which also result in optimality. All the distributions in this class have the property that the higher order moments can be expressed in terms of just their mean and variance.

2.7 Choosing an efficient portfolio

As mentioned above, indifference curves join points of equal expected utility in expected return-standard deviation space, *ie* portfolios that an individual is indifferent between. Note that it is *expected utility* because we are considering situations involving uncertainty.

A series of indifference curves (curves which join all outcomes of equal utility) can be plotted in expected return–standard deviation space.

Portfolios lying along a single curve all give the same value of expected utility and so the investor would be indifferent between them.

By combining the investor's indifference curves with the efficient frontier of portfolios, we can determine the investor's optimal portfolio, *ie* the portfolio that maximises the investor's expected utility.

Utility is maximised by choosing the portfolio on the efficient frontier at the point where the frontier is at a tangent to an indifference curve.

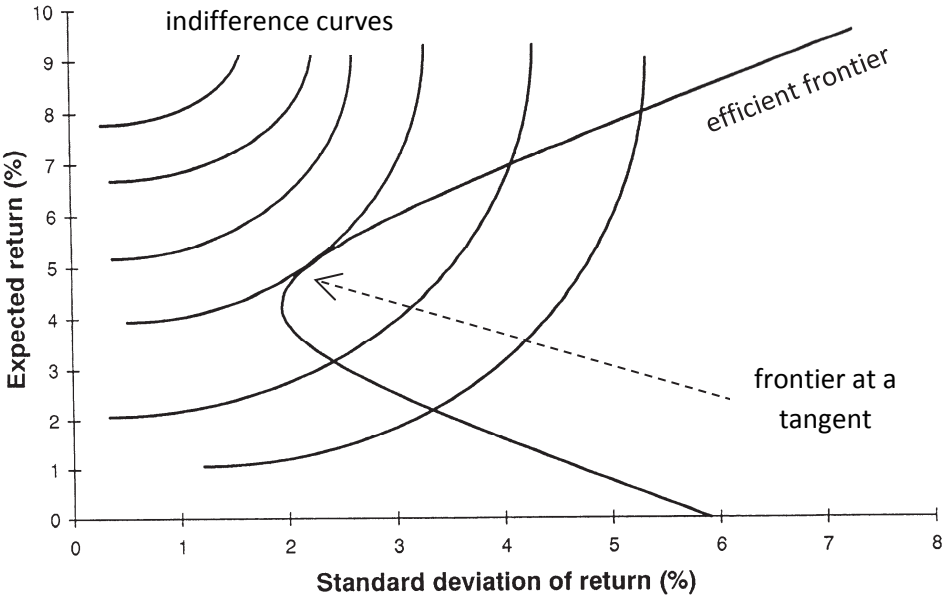

Figure 18.3

Question

Why do the investor's indifference curves slope upward? What determines their gradient?

Solution

The investor's indifference curves slope upwards because the investor is assumed to be risk-averse and prefer more to less. Consequently, additional expected return yields extra utility, whereas additional risk reduces utility. Thus, any increase (decrease) in risk/standard deviation must be offset by an increase (decrease) in expected return in order to maintain a constant level of expected utility.

The gradient of the indifference curves is determined by the degree of the investor's risk aversion. The more risk-averse the investor, the steeper the indifference curves – as the investor will require a greater increase in expected return to offset any extra risk.

For quadratic utility functions the process described above produces optimal portfolios whatever the distribution of returns, because expected utility is uniquely determined if we know the mean and variance of the distribution.

If it is felt that the assumptions leading to a two-dimensional mean-variance type portfolio selection model are inappropriate, it is possible to construct models with higher dimensions. For example, skewness could be used in addition to expected return and a dispersion measure.

Here, 'dispersion measure' just means a measure of dispersion about the mean, such as the standard deviation or variance.

It would then be necessary to consider an efficient *surface* in three dimensions rather than an efficient frontier in two. Clearly, the technique can be extended to more than three dimensions.

Although such models have been constructed, they do not appear to be widely used. It is doubtful whether the additional mathematical complexity, input data requirements and difficulty of interpretation in a pragmatic way, are compensated by real improvements in value added.

2.8 Benefits of diversification

Recall, from Section 2.4 above, that the variance of a portfolio is:

$$V = var[R_P] = \sum_i \sum_j C_{ij} x_i x_j$$

This expression can be rewritten as:

$$V = \sum_i x_i^2 V_i + \sum_i \sum_{j, j \neq i} C_{ij} x_i x_j$$

Recall also that, the lower (*ie,* closer to zero) the covariance between security returns, the lower the overall variance of the portfolio. This means that the variance of a portfolio can be reduced, by investing in securities whose returns are uncorrelated or, equivalently, investing in independent assets.

Where all assets are independent, the covariance between them is zero and the formula for variance becomes:

$$V = \sum_i x_i^2 V_i$$

If we assume that equal amounts are invested in each asset, then with N assets the proportion invested in each is $1/N$. Thus:

$$V = \sum_i (1/N)^2 V_i = (1/N) \sum_i (1/N) V_i = \frac{\bar{V}}{N}$$

where $\bar{V}$ represents the average variance of the stocks in the portfolio. As N gets larger and larger, the variance of the portfolio approaches zero. This is a general result – if we have enough *independent* assets, the variance of a portfolio of these assets approaches zero.

In other words, a lower variance, *ie* lower risk, can be achieved by diversification.

In general, we are not so fortunate. In most markets, the correlation coefficient and the covariance between assets is positive.

In these markets, the risk on the portfolio cannot be made to go to zero, but can still be much less than the variance of an individual asset. With equal investment, the proportion invested in any one asset x_i is $1/N$ and the formula for the variance of the portfolio becomes:

$$V = \sum_i (1/N)^2 V_i + \sum_i \sum_{j, j \neq i} (1/N)(1/N) C_{ij}$$

Factoring out $1/N$ from the first summation and $(N-1)/N$ from the second yields:

$$V = (1/N) \sum_i (1/N) V_i + \frac{(N-1)}{N} \sum_i \sum_{j, j \neq i} \frac{C_{ij}}{N(N-1)}$$

Replacing the variances and covariances in the summation by their averages ($\bar{V}$ and $\bar{C}$), we have:

$$V = \frac{\bar{V}}{N} + \frac{N-1}{N} \bar{C}$$

The contribution to the portfolio variance of the variances of the individual securities goes to zero as N gets very large. However, the contribution of the covariance terms approaches the average covariance as N gets large.

So, as the number of assets in the portfolio is increased, the variance of the portfolio return gets closer to the average covariance of return between the pairs of assets in that portfolio.

The individual risk of securities can be diversified away, but the contribution to the total risk caused by the covariance terms cannot be diversified away.

Chapter 18 Summary

Monitoring and controlling risk

Investors need systems to monitor and control the different forms of financial risk:

- *Market risk* is the risk relating to changes in the value of the portfolio due to movements in the market value of the assets held. It can be measured using *Value at Risk* (the possible loss in the value of the fund, with a probability of *p*% over a time period of *t*) and controlled by regular modelling and reporting.
- *Credit risk* is the risk that a counterparty to an agreement will be unable or unwilling to fulfil their obligations. It can be controlled:
  - by limiting the creditworthiness of the counterparties
  - by limiting the total exposure to each counterparty.
  - using credit derivatives.
- *Operational risk* is the risk of loss due to fraud or mismanagement within the fund management organisation itself. It can be controlled by appropriate internal reporting and by separation of front office and back office functions.
- *Liquidity risk* is the risk of not having sufficient cash to meet operational needs at all times.

Liquidity risk elasticity (LRE) considers the impact of changes in market conditions. The process consists of two steps:

1. Calculate the present value of assets and liabilities using the 'cost of funds' rate as the discount rate.
2. Measure the change in the market value of the institution's equity (LRE) from a change in the cost of funds (due to an increase in the risk-premium paid to raise money).

If the LRE is zero, the institution has zero liquidity risk (by this measure). If the LRE is sharply negative, it will pay the institution to shorten the maturity of its assets and lengthen the maturity of its liabilities, thereby increasing liquidity.

- *Relative performance risk* is the risk of under-performing comparable institutional investors. It can be measured and controlled in the same way as market risk.

Portfolio theory

Portfolio theory enables the investor to identify its optimal portfolio – the one that maximises expected utility as a function of the mean and variance of investment returns.

The assumptions underlying portfolio theory are as follows:

- All expected returns, variances and covariances of pairs of assets are known.
- Investors make their decisions purely on the basis of expected return and variance.
- Investors are non-satiated.
- Investors are risk-averse.
- There is a fixed single-step time period.
- There are no taxes or transaction costs.
- Assets may be held in any amounts, *ie* short-selling, infinitely divisible holdings, no maximum investment limits.

The *opportunity set* is the set of combinations of means and variances that the investor is able to obtain by constructing portfolios containing the available securities.

A portfolio is *efficient* if there is no other portfolio with either a higher mean and the same or lower variance, or a lower variance and the same or higher mean. The *efficient frontier* is the set of efficient portfolios in $E-V$ space.

Indifference curves join points of equal expected utility in $E-V$ space.

The *optimal portfolio* is the portfolio that maximises the investor's expected utility as a function of the mean and variance of investment returns.

Portfolio theory can be used to explain how individual risk can be diversified away.

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Solving problems

Syllabus objectives

9.1 Analyse hypothetical examples and scenarios in relation to the management of investments.

- 9.1.1 Propose solutions and actions that are appropriate to the given context, with justification where required.
- 9.1.2 Suggest possible reasons why certain actions have been chosen.
- 9.1.3 Assess the implications of actions within a given scenario.
- 9.1.4 Discuss the advantages and disadvantages of suggested actions, taking into account different perspectives.

0 Introduction

This chapter has no new material, and is very brief. The Core Reading is as follows:

The examiners will expect candidates to be able to apply the knowledge and understanding they have developed through the study of the SP5 Core Reading to produce coherent solutions and actions in relation to the management of investments.

In particular students are expected to be able to combine ideas across the Core Reading, and apply them to scenarios proposed by the examiners.

One of the main differences between SP-subjects and earlier subjects is that students may be presented with more open-ended problems, perhaps relating to events that have occurred in the real world at some point in the months or years prior to the exam. The subject matter should relate to the Subject SP5 syllabus objectives, but may only loosely relate to any Core Reading contained in this course.

The objective is for well-prepared students to be able to show their 'higher-order' skills by successfully:

- analysing each scenario,
- identifying a solution or a range of solutions, and
- discussing the benefits and drawbacks of each.

(Coming to a definitive recommendation as such is less likely to be an expectation.)

1 Example of solving problems in Subject SP5

We have added an example of a typical 'higher order skills' question from a past exam to illustrate some of the additional skills that are required to pass SP-subject exams.

Question – Subject ST5, April 2013, Question 7

You are a new provider of an individual defined contribution pensions savings product with a simplified investment approach sold via the internet. The product charges a total management fee of 50bps *pa* (all assets are directly managed) and there are only two fund choices:

- a growth fund, and
- an inflation-linked annuity matching fund (at retirement it is mandatory to use pension savings to buy an inflation-linked annuity).

The growth fund invests in a diversified range of asset classes with a benchmark asset allocation of:

- 40% equity
- 20% alternative asset classes (property, private equity, infrastructure)
- 20% investment grade bonds
- 20% high yield bonds and asset backed credit.

Individual contributors can either select their chosen mix of growth fund and annuity fund units, or opt for a 'lifestyling' approach that phases from the growth fund into the annuity fund based on the chosen retirement date. Individuals are permitted to transfer to another provider with no penalty.

You are confident that the low fee scale, the simplified approach and the attractions of diversified investment strategies will permit the rapid growth of market share and assets under management.

(i) Describe the advantages and disadvantages of such an approach from the investor's perspective. [8]

(ii) Describe the specific challenges for the provider in managing the underlying investments as assets under management grow. [8]

[Total 16]

Question analysis

Before reading our analysis of the question (above), try writing out a solution for yourself.

When approaching such a question we should use:

- all the information that the Examiner has given us
- any knowledge that we may have on 'lifestyling' (*eg* from experience in the insurance industry)
- the knowledge obtained by studying Subject SP5 (*eg* concerning infrastructure assets, private equity investments, asset-backed securities, property and high-yield bonds), and
- good exam technique – to maximise the number of marks we obtain from the information we have.

Starting with the specific information that the Examiner has included in the question, we might firstly look at the following sentence:

> 'You are confident that the low fee scale, the simplified approach and the attractions of diversified investment strategies will ...'

This gives us three things to talk about straight away:

- this is a simple product.
- the product has low fees compared to other products in the marketplace
- the product has a diversified asset portfolio, invested across a range of asset classes.

In addition, the question tells us that we are a 'new provider' and that the fee is 50 bps (1 bp = 1 basis point = 0.01%) per annum. These should also be addressed.

'Lifestyling' is, as described in the question, an approach whereby the investor allows the life office to automatically switch investment from a risky, volatile fund, into a fund that more accurately matches the liability to purchase an annuity at retirement. When this concept is mixed with the fact that the client deals through the internet, and therefore gets no advice, the concept of mis-selling may come to mind. Lifestyling is a popular approach however, and investors often prefer not to have to make the trickier investment decisions themselves.

Considering the diversified growth portfolio, it is clear that it contains a reasonable number of specialist asset classes that require expertise to manage, and that investors could not normally get exposure to through other means. From our study of Subject SP5 we know that many of these asset classes are illiquid, have lock-in periods, minimum investment sizes, and even capacity constraints for investors that wish to increase (or decrease) exposure rapidly. So considering such problems in our answer is important, whilst remembering that the question stipulates that the solution should be 'from an investor's perspective.

Part (ii) focuses on 'growth' in fund size. We might imagine here what would happen if the fund *really* takes off, and begins to double in size each year. What problems would be encountered?

It is important to notice that the benchmark has a high percentage in asset classes that can be very illiquid (property, private equity, high yield bonds, *etc*). This is where a lot of the problems will lie. When funds grow large, they have the capacity to shrink by a larger amount.

The question also gives a further clue in that it states 'Individuals are permitted to transfer to another provider with no penalty'. From knowledge of the industry, we should be aware that this is unusual, and potentially quite risky. If a fund grows large, there may be some very large investors who, in the absence of a penalty, may choose to switch out at a moment's notice.

If you haven't already, try writing out a solution before reading on.

Solution

(i) ***Advantages and disadvantages from investor's perspective***

Advantages

This pensions savings product is quite *simple* in its operation, which:

- makes it harder for investors to make *poor savings* choices, and
- means that investors may need less *advice* going forward to manage their pension plan.

These are both especially true for investors that have *less financial expertise* and may be overwhelmed by the large ranges of funds offered in the market.

The management fee is 0.5%, which is quite low. This is an advantage for investors.

The cost advantages of the product are due to *low distribution costs* and the elimination of the extra layer of fees associated with high-fee investment funds like specialist funds and hedge funds.

The growth fund still allows access to expertise in terms of *asset allocation* at both an individual and asset class level. This expertise may also extend to *stock and sector selection*.

The growth fund is well *diversified*, investing in many different asset classes. This *reduces the risk* for an investor wishing to safeguard the value of their retirement savings.

The product may offer *access to some asset classes* that are not available in alternative products on the market.

Disadvantages

The simplified nature of the product means that there is *less choice* for an investor who prefers to invest with a more traditional provider operating a number of different investment funds.

There is considerable *loss of control* in terms of asset selection, as this is fixed in the benchmark, and strong dependency on the expertise of the asset manager to make tactical asset allocation (TAA) decisions correctly.

This dependency is exacerbated by the benchmark of the growth fund.

The investor will need to trust that the provider has *stock selection* ('alpha') expertise in every investment class covered by the benchmark. This might be particularly a problem in the more specialist asset classes, such as property, private equity and infrastructure.

There is no historical data for this product for the investor to see, so there isn't necessarily a guarantee that the low management charge can be *maintained* over time.

Because the product is new, the costs of maintaining it may have been *underestimated*.

There may be problems with the investment approach needing to be *scaled up.* Some asset classes have maximum investment amounts, which may become a problem if the product becomes really popular.

(ii) ***Challenges in managing the underlying investments as assets grow***

The growth fund invests in a diversified range of asset classes, so this will make it easier to manage than otherwise.

However, this does rely on a good understanding of liquidity requirements, inflows and outflows.

For example, a significant outflow may cause a problem when we consider the liquidity of the portfolio.

Alternative asset classes, high yield bonds and asset backed credit make up 40% of the benchmark portfolio and such assets may be difficult to sell quickly at a favourable price.

This may lead to the need to sell equities and perhaps corporate bonds instead, which will shift the asset allocation more towards the illiquid assets.

Individuals are permitted to transfer to another provider with no penalty. This may be a problem in terms of allocating their return when they leave, because the value of investments such as private equity and infrastructure will not be known at all times.

A significant inflow may cause a problem in terms of assets being held as cash pending identification of suitable investment opportunities.

Again, this is most likely to be the case for the alternative asset classes, high yield bonds and asset-backed credit.

There may be a tendency to instead invest in equities and corporate bonds in this situation, which shifts the asset allocation away from the benchmark.

The investment manager will need to take particular care over the scaling up staff resources as the assets under management grow. This is particularly important in new asset classes, where a great deal of skill is required to generate alpha or to make tactical switches.

Summary

As you can see, some questions at this level of the exams can require real problem solving skills, and some market knowledge.

Many of the skills and exam techniques required are similar to the ones used in earlier subjects (especially Subject CP1), in terms of:

- developing points
- describing the ideas that you have in greater detail
- including examples to assist the explanation of a concept and add clarity
- using some broad idea generators (*eg* categories of risks).