Subject CM2

Corrections to 2019 study material

This document contains details of any errors and ambiguities in the Subject CM2 study materials for the 2019 exams that have been brought to our attention. We will incorporate these changes in the study material each year. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses. If you have any such comments on this course please email them to CM2@bpp.com.

This document was last updated on **12 August 2019**.

1 Paper A Course Notes

Chapter 2

Correction added on 12 August 2019

Page 31

The premium in the penultimate sentence should be £8,333.33, *ie*:

This individual would be willing to pay up to £8,333.33 for insurance that covers any loss.

Chapter 9

Correction added on 1 April 2019

Page 6

The final equation requires an additional bracket:

$$\begin{aligned} Cov(W_s, W_t) &= E[(W_s - E[W_s])(W_t - E[W_t])] \\ &= E[W_s(W_s + (W_t - W_s))] \end{aligned}$$

Chapter 10

Corrections added on 1 April 2019

Page 7

The strengths of the inequalities need changing so that the sentence towards the bottom of the page becomes:

Assuming n is large, $f(t)$ can be approximated by $f(t_{i-1})$ where $t_{i-1} \le t < t_i$.

Page 9

The integrand needs to be evaluated at time t_{i-1} rather than time t_i, therefore the Ito integral should be defined as:

$$\int_0^T f(W_t, t)dW_t = \lim_{n\to\infty} \sum_{i=1}^{n} f(W_{t_{i-1}}, t_{i-1})\left(W_{t_i} - W_{t_{i-1}}\right)$$

Page 37

The third bullet point in the Ito integrals section should be replaced with:

- have zero mean and a variance of $\int_0^T E\left[f^2(W_t, t)\right]dt$.

Chapter 17

Corrections added on 1 April 2019

Page 9

The general formula at the top of the page shouldn't be limited to only dividend-paying shares. In the first sentence remove the words "on a dividend-paying share".

Page 33

Question 17.2(i)(b) should ask for only *one* "real-world quantity this is commonly modelled using such as process" and the marks available for part (i) should be reduced to [2].

Page 39

Solution 17.2(i)(b) should no longer refer to "currency exchange rates" and the marks available for part (i)(b) should be reduced to [1].

Chapter 18

Corrections added on 1 April 2019

Page 20

Remove the words "r_t will never hit zero" from the parenthesis.

Page 35

The definition of the instantaneous forward rate should be:

$$f(t,T) = \lim_{T \to S} f(t,T,S) = -\frac{\partial}{\partial T} \log P(t,T)$$

Chapter 20

Corrections added on 1 April 2019

Page 12

The reference to "Section 0" should be "Section 2.2".

Page 26

The lower bound section of the solution to this question should be removed, ending the solution with "the correct value of approximately 0.000026."

Page 31

The sentence describing Figure 7 should say:

Also shown in Figure 7 are $\psi(15)$ (dotted line) and $\exp\{-15R\}$ (solid line) for this portfolio.

Page 33

The two sentences describing Figure 8 should say:

Figure 8 shows values of $\psi(U,t)$ for $0 \le t \le 500$ and for three values of the initial surplus, $U = 15, 20,$ and 25. The premium loading factor is 0.1 as in Figure 7. For $U = 15$ the graph of $\psi(U,t)$ is as in Figure 7.

The caption to Figure 8 should say:

Figure 8 – $\psi(U,t)$ for different values of U

The reference to “Figure 4” should say “Figure 7”.

Page 34

Item (iii) shouldn’t refer to the case of $\theta = \mathbf{0.1}$, only of $\theta = \mathbf{0.2}$ **and 0.3**.

Page 38

Subscript “1”s are missing in four locations. A replacement page has been added to the end of this document.

Page 48

After equation (5.5) the sentence should say:

The right hand side is based on the MGF of the net claim amounts which have an $Exp(0.1/\alpha)$ distribution.

Page 52

The reference to “Section 0” should be “this section” instead.

Page 57

The definition of c should be:

$$c = (1+\theta)E[S(1)]$$

Page 59

The Summary box on this page should be deleted.

2 Revision Notes

Booklet 8

Correction added on 26 July 2019

Page 70

The first bullet point should be:

- will generally reduce the overall claim volatility, which will reduce the probability of ruin. The *lower* the retention limit the greater the reduction.

The unit of time is taken to be one year. The only difference between these risks is that twice as many claims are expected each year under Risk 1. This is reflected in the two premiums.

Consider Risk 2 over a new time unit equivalent to two years. Then the distribution of aggregate claims and the premium income per unit time are now identical to the corresponding quantities for Risk 1. Hence, the probability of ruin over an infinite time span is the same for both risks.

The solid line in Figure 12 shows an outcome of the surplus process for Risk 1 when $\theta = 0.1$. The dotted line shows the same surplus process when the unit of time is two years. This illustrates that any outcome of the surplus process that causes ultimate ruin for Risk 1 will also cause ultimate ruin for Risk 2. There is thus no difference in the probability of ultimate ruin for these two risks. It is only the time (in years) until ruin that will differ. Measuring times in years, the probability of ruin by time 1 for Risk 1 is the same as the probability of ruin by time 2 for Risk 2. This explains why Figures 9 and 11 show the same functions. For example, the value of $\psi(15,10)$ when $\lambda = 50$ (Figure 11) is the same as the value of $\psi(15,500)$ when $\lambda = 1$ (Figure 9).

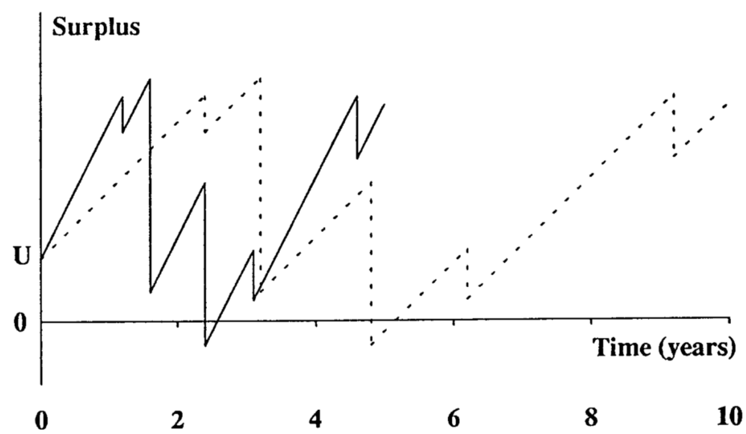

Figure 12

Point (iii) in Section 4.5 will now be investigated, where it was noted that values of $\psi(15,t)$ were more or less constant for values of t greater than 150 when $\theta = 0.2$ and 0.3. In particular, the situation will be considered when the premium loading factor is 0.2.

Consider a second aggregate claims process, which is the same as the process considered throughout this section except that its Poisson parameter is 150 and not 1. (This second process is really identical to the original one; all that has happened is that the time unit has been changed.) Use ψ^* to denote ruin probabilities for the second process and ψ to denote, as before, ruin probabilities for the original process. The change of time unit means that for any $t \geq 0$:

$$\psi^*(U,t) = \psi(U,150t)$$

but it has no effect on the probability of ultimate ruin (put $t = \infty$ in the relationship above) so that:

$$\psi^*(U) = \psi(U)$$

The point made in (iii) above was that:

$$\psi(15,150) \approx \psi(15)$$

From this and the previous two relations it can be seen that:

$$\psi^*(15,1) \approx \psi^*(15)$$

In words this relation says that for the second process, starting with initial surplus 15, the probability of ruin within one time period is almost equal to (actually a little less than) the probability of ultimate ruin. This conclusion depends crucially on the fact that $\psi^*(15,1)$ is a continuous time probability of ruin. To see this, consider $\psi_1^*(15,1)$, which is just the probability that for the second process the surplus at the end of one time period is negative. $\psi_1^*(15,1)$ can be calculated approximately by assuming that the aggregate claims in one time period, which will be denoted $S^*(1)$, have a normal distribution. Recall that individual claims have an exponential distribution with mean 1 and that the number of claims in one time period has a Poisson distribution with mean 150. From this:

$$E\left[S^*(1)\right] = 150 \qquad \text{and} \qquad Var\left(S^*(1)\right) = 300$$

These are calculated as $\lambda m_1 = 150 \times 1 = 150$ and $\lambda m_2 = 150 \times 2 = 300$, where $E\left[X^2\right] = 2$ for an $Exp(1)$ distribution.

Now, using tables of the normal distribution:

$$\psi_1^*(15,1) = P\left[S^*(1) > 15 + 1.2 \times 150\right]$$

$$= P\left[(S^*(1) - 150)/17.32 > 45/17.32\right]$$

$$\approx 0.005$$

Recall that if $X \sim N(\mu, \sigma^2)$, then $Z = \dfrac{X-\mu}{\sigma}$ has a standard normal distribution, *ie* $Z \sim N(0,1)$. Probabilities for this distribution can be looked up in the *Tables*.

From Figure 9 it can be seen that the value of $\psi(15,150)$, and hence of $\psi^*(15,1)$, is about 0.07 which is very different from the (approximate) value of the discrete time probability of ruin $\psi_1^*(15,1)$ calculated above.

4.7 Concluding remarks

When individual claim amounts are exponentially distributed with mean 1, first note that if $\theta = 0$, then $\psi(U) = 1$ irrespective of the value of U.

We're thinking here of substituting $\theta = 0$ into equation 4.4.

This result is in fact true for any form of $F(x)$ (it trivially follows that if $\theta < 0$, then $\psi(U) = 1$). In other words a positive premium loading is essential if ultimate ruin is not to be certain.