CMP Upgrade 2019/20

Subject CM1

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus objectives, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2019 CMP to make it suitable for study for the 2020 exams. It includes replacement pages and additional pages where appropriate. Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our 2020 *Student Brochure* for more details.

This CMP Upgrade contains:

- all significant changes to the Syllabus objectives and Core Reading
- additional changes to the ActEd Course Notes and Assignments that will make them suitable for study for the 2020 exams.

1 Changes to the Syllabus objectives

This section contains all the ***non-trivial*** changes to the Syllabus objectives.

Syllabus objective 1 has been renamed ‘The basics of modelling’ (previously ‘Data and basics of modelling’).

Syllabus objective 1.1 on data analysis has been removed (including sub-objectives 1.1.1 to 1.1.4).

As a result of the removal of Syllabus objective 1.1, Syllabus objectives 1.2 and 1.3 have been renumbered as 1.1 and 1.2.

2 Changes to the Core Reading and ActEd text

This section contains all the ***non-trivial*** changes to the Core Reading and ActEd text.

Chapter 1

Following the deletion of Syllabus objective 1.1, this chapter has been removed from the course.

All subsequent chapter numbers in the 2020 version of the course have been reduced by 1.

For the purposes of describing the changes made, in this document we have retained the 2019 chapter numbering, although any replacement pages attached feature the new (2020) chapter numbering.

Chapter 2

Page 5

The following sentence of Core Reading has been added in the 'Data' section:

The analysis of data is covered in more detail in Subject CS1.

Chapter 7

Section 2.2

This section has been expanded to include a formula for the present value of a continuous payment stream at time 0. Replacement pages 13-16 are provided.

Chapter 13

Page 5

The description of the solution on this page has been expanded. Replacement pages 5-6 have been provided.

Page 18

In the second paragraph of the question towards the top of this page, the words 'for £110' have been deleted from the first sentence.

This sentence should read 'An investor, who is subject to tax at 25% on income and capital gains, purchases £100 nominal of this stock.'

Chapter 14

Page 11

In the solution to part (ii) of the question that starts on page 10, the continuous-time forward rate quoted should be $F_{5,10}$ throughout, not $F_{5,15}$.

The equation of value for the 15-year bond has been corrected to:

$$30 = 100e^{-5Y_5}\, e^{-10F_{5,10}}$$

The last formula in the solution has been corrected to:

$$30 = 70e^{-10F_{5,10}}$$

Section 3.4

This section has been expanded to include additional description of the par yield. Replacement pages 19-20 are provided.

Chapter 21

Solutions 21.1 and 21.4

These solutions have been amended. Replacement pages 47-50 are provided. Note that you will need to retain the old page 50 to preserve the start of the solution to question 9.

Page 54

In the middle of the page, the EPV of the claim expense on death has been corrected to:

$$1{,}000\bar{A}^{\;1\,@4\%}_{60:\overline{5}|} \approx 1{,}000 \times 1.04^{½} \times \left[A_{60:\overline{5}|} - \frac{D_{65}}{D_{60}} \right]$$

$$= 1{,}000 \times 1.04^{½} \times \left[0.82499 - \frac{689.23}{882.85} \right] = 45.180$$

The corrected figure of 45.180 is used in place of 44.302 in the subsequent calculation of the gross premium prospective reserve, to produce a final answer of 234,880.

Chapter 24

Solutions 24.3 and 24.4

These solutions have been expanded. Replacement pages 29-32 are provided. Note that you will need to retain the old page 32 to preserve the start of the solution to question 6.

Chapter 25

Pages 25 and 31

In the tables in the questions on these pages, the column headings $(aq)_x^d$ and $(aq)_x^w$ have been corrected to $(ad)_x^d$ and $(ad)_x^w$.

Chapter 28

Section 2.4 has been added, which includes an example of a full unit-linked profit test. Replacement pages 17-26 are provided. These replace the old pages 17-18 only.

3 Changes to the X Assignments

Assignment X1

Question X1.5 has been changed, as follows:

Question

Describe the cashflows experienced by a term assurance policyholder. [3]

Solution

The cashflows for a term assurance policyholder will be a series of negative cashflows (*ie* premium payments) throughout the specified term or until earlier death (or one negative cashflow at inception if the premium is paid on a lump sum basis)… [1]

…followed by a large positive cashflow (*ie* the benefit payment) payable on death, if death occurs before the end of the term. [1]

If the policyholder survives to the end of the term there is no positive cashflow. [1]

[Total 3]

Assignment X4

Question X4.5 has been changed to make it clear that the answer should be written in terms of assurance functions, and the solution has been updated.

Question

A joint life annuity of 1 *pa* is payable continuously to lives currently aged x and y while both lives are alive. The present value of the annuity payments is expressed as a random variable, in terms of the joint future lifetime of x and y.

Derive, and simplify as far as possible, expressions for the expected present value and the variance of the present value of the annuity, in terms of assurance functions. [5]

Solution

The present value random variable for this annuity is:

$$\bar{a}_{\overline{T_{xy}}|} \quad \text{or} \quad \bar{a}_{\overline{\min\{T_x,T_y\}}|}$$ [½]

The expected present value is:

$$E\left(\bar{a}_{\overline{T_{xy}}|}\right)=E\left(\frac{1-v^{T_{xy}}}{\delta}\right)=\frac{1-E\left(v^{T_{xy}}\right)}{\delta}$$ [½]

Now:

$$E\left(v^{T_{xy}}\right)=\bar{A}_{xy}$$ [½]

So:

$$E\left(\bar{a}_{\overline{T_{xy}|}}\right)=\frac{1-\bar{A}_{xy}}{\delta}$$ [½]

The variance of the present value random variable is:

$$\text{var}\left(\bar{a}_{\overline{T_{xy}|}}\right)=\text{var}\left(\frac{1-v^{T_{xy}}}{\delta}\right)=\frac{1}{\delta^2}\text{var}\left(v^{T_{xy}}\right)=\frac{1}{\delta^2}\left[E\left(v^{2T_{xy}}\right)-\left[E\left(v^{T_{xy}}\right)^2\right]\right]$$ [1½]

Now:

$$E\left(v^{2T_{xy}}\right)={}^2\bar{A}_{xy}$$ [½]

where the superscript of 2 to the left of the assurance symbol indicates that the assurance is evaluated using twice the standard force of interest, which is equivalent to evaluating using the rate of interest $i'=(1+i)^2-1$. [½]

So the variance of the present value random variable is:

$$\text{var}\left(\bar{a}_{\overline{T_{xy}|}}\right)=\frac{1}{\delta^2}\left[{}^2\bar{A}_{xy}-\left(\bar{A}_{xy}\right)^2\right]$$ [½]

[Total 5]

4 Changes to the Y Assignments

There have been some significant changes made to the format and structure of the questions in the Y Assignments.

You should use the 2020 versions in your exam preparation.

5 Other tuition services

In addition to the CMP you might find the following services helpful with your study.

5.1 Study material

We also offer the following study material in Subject CM1:

- Flashcards
- Revision Notes
- ASET (ActEd Solutions with Exam Technique) and Mini-ASET
- Mock Exam and AMP (Additional Mock Pack).

For further details on ActEd's study materials, please refer to the 2020 *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

5.2 Tutorials

We offer the following (face-to-face and/or online) tutorials in Subject CM1:

- a set of Regular Tutorials (lasting five full days)
- a Block (or Split Block) Tutorial (lasting five full days)
- a Preparation Day for the computer-based exam
- an Online Classroom.

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **www.ActEd.co.uk**.

5.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to the 2020 *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

5.4 Feedback on the study material

ActEd is always pleased to get feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course please send them by email to **CM1@bpp.com**.

This shows that we can think of the factors $(1+i)^n$ and v^n as a way of adjusting payments to a different point on the timeline.

If the present value of a series of definite payments at a particular date is X, then:

- the accumulated value at a date *n* years later is $X(1+i)^n$
- the present value at a date *n* years earlier is Xv^n.

Note that *n* does not have to be a whole number in these formulae.

Question

Under its current rental agreement, a company is obliged to make annual payments of £7,500 for the building it occupies. Payments are due on 1 January 2020, 1 January 2021 and 1 January 2022. The nominal rate of interest is 8% per annum, convertible quarterly.

Calculate the value of these rental payments on:

(i) 1 January 2019

(ii) 1 January 2018

(iii) 1 July 2033

Solution

A nominal rate of interest of 8% *pa* convertible quarterly is equivalent to a quarterly effective interest rate of $\frac{8\%}{4} = 2\%$.

(i) Working in quarters, the value of the rental payments on 1 January 2019 is:

$$7{,}500(v^4 + v^8 + v^{12}) = 7{,}500(1.02^{-4} + 1.02^{-8} + 1.02^{-12}) = £19{,}243.72$$

(ii) Since 1 January 2018 is 1 year (= 4 quarters) before 1 January 2019, the value of the rental payments on 1 January 2023 is:

$$19{,}243.72 \times 1.02^{-4} = £17{,}778.22$$

(iii) Since 1 July 2033 is 14.5 years (= 58 quarters) after 1 January 2019, the value of the rental payments on 1 July 2033 is:

$$19{,}243.72 \times 1.02^{58} = £60{,}687.46$$

Alternatively, we could first calculate the effective annual rate, i, as $1.02^4 - 1 = 8.243216\%$*, and then work in years. So, for example, the value in part (i) is:*

$$7{,}500(v + v^2 + v^3) = £19{,}243.72 \text{ where } v = 1/1.08243216$$

2.2 Payment streams

In Section 1.2, we saw that the present value at time 0 of a continuous payment stream received from time 0 to time T, where the rate of payment at time t is $\rho(t)$, is given by:

$$PV_{t=0} = \int_0^T v(t)\rho(t)dt$$

If the continuous payment stream is received from time a to time b, then this formula becomes:

$$PV_{t=0} = \int_a^b v(t)\rho(t)dt$$

Intuitively, we can obtain this formula by first considering the payment at time t, which is at a rate of $\rho(t)$. This payment needs to be discounted back to time 0 and the appropriate discount factor is $v(t)$. Finally, we need to add together all the present values of the payments at the different times. Since we are receiving payments continuously, we integrate these present values between the limits a and b, *ie* the times between which the payment comes in.

We can also work out the present value of a continuous payment stream at a time other than time 0. Suppose we wanted to calculate the value of the same payment stream as at the start of the payment period, *ie* at time a rather than at time 0. If the force of interest at time s is $\delta(s)$, this present value is:

$$PV_{t=a} = \int_a^b \rho(t)\exp\left(-\int_a^t \delta(s)ds\right)dt$$

We can think of this formula in the same way as that given above, except that instead of discounting payments back to time 0 using $v(t)$, we need to discount them back to time a, which can be done using the discount factor $\exp\left(-\int_a^t \delta(s)ds\right)$.

Question

A continuous payment stream is paid at rate $e^{-0.03t}$ from time $t=0$ to time $t=10$.

Calculate the present value of this payment stream at time $t=0$, given that the force of interest over this time period is 0.04 *pa*.

Solution

The payment stream starts at time 0 and finishes at time 10, so we can set $a=0$ and $b=10$. Using these values along with $\rho(t)=e^{-0.03t}$ and $\delta(t)=0.04$ gives:

$$PV_{t=0}=\int_0^{10} e^{-0.03t}e^{-\int_0^t 0.04ds}\,dt=\int_0^{10} e^{-0.03t}e^{-[0.04s]_0^t}\,dt=\int_0^{10} e^{-0.03t}e^{-0.04t}\,dt$$

This integral simplifies and is evaluated as follows:

$$PV_{t=0}=\int_0^{10} e^{-0.07t}\,dt=\left[-\frac{1}{0.07}e^{-0.07t}\right]_0^{10}=\frac{1}{0.07}(1-e^{-0.7})=7.192$$

We can also consider the accumulated value of the continuous payment stream paid at a rate of $\rho(t)$ from time *a* to time *b*, during which time the force of interest is $\delta(t)$. The accumulated value at time *b* of this payment stream is:

$$AV_{t=b}=\int_a^b \rho(t)\exp\left(\int_t^b \delta(s)ds\right)dt$$

Intuitively, we can obtain this formula by first considering the payment at time *t*, which is at a rate of $\rho(t)$. This payment needs to be accumulated to time *b* and the accumulation factor is $\exp\left(\int_t^b \delta(s)ds\right)$ in terms of the force of interest. Finally, we need to add together all the accumulated values of the payments at the different times. Since we are receiving payments continuously, we integrate these accumulated values between the limits *a* and *b, ie* the times between which the payment comes in.

Question

The force of interest at time t, where $0\le t\le 10$, is given by $\delta(t)=0.07$.

Calculate the accumulated value at time 10 of a payment stream, paid continuously from time 5 to time 10, under which the rate of payment at time t is $10e^{0.05t}$.

Solution

The payment stream starts at time 5 and finishes at time 10, so we can set $a=5$ and $b=10$. Using these values along with $\rho(t)=10e^{0.05t}$ and $\delta(t)=0.07$ gives:

$$AV_{t=10}=\int_5^{10} 10e^{0.05t}e^{\int_t^{10} 0.07ds}dt=\int_5^{10} 10e^{0.05t}e^{[0.07s]_t^{10}}\,dt=\int_5^{10} 10e^{0.05t}\,e^{0.7-0.07t}dt$$

This integral simplifies and is evaluated as follows:

$$AV_{t=10} = 10e^{0.7}\int_5^{10} e^{-0.02t}\,dt = 10e^{0.7}\left[\frac{e^{-0.02t}}{-0.02}\right]_5^{10} = 10e^{0.7}\left(\frac{e^{-0.2}-e^{-0.1}}{-0.02}\right) = 86.699$$

One result that can be useful here is the chain rule for differentiation expressed in integral form. The chain rule applied to the exponential of a function tells us that:

$$\frac{d}{dt}e^{f(t)} = f'(t)e^{f(t)}$$

Integrating both sides, we have:

$$\int_a^b f'(t)e^{f(t)}\,dt = \left[e^{f(t)}\right]_a^b$$

Question

The force of interest at time t is given by:

$$\delta(t) = 0.01t + 0.04 \qquad 0 \le t \le 5$$

Calculate the present value at time 0 of a payment stream, received continuously from time 0 to time 5, under which the rate of payment at time t is $0.5t + 2$.

Solution

Here $a = 0$, $b = 5$ and $\rho(t) = 0.5t + 2$, so we have:

$$\begin{aligned}
PV_{t=0} &= \int_0^5 (0.5t+2)\exp\left(-\int_0^t (0.01s+0.04)\,ds\right)dt \\
&= \int_0^5 (0.5t+2)\exp\left(-\left[0.005s^2+0.04s\right]_0^t\right)dt \\
&= \int_0^5 (0.5t+2)\exp\left(-\left[0.005t^2+0.04t\right]\right)dt
\end{aligned}$$

Now, using the general result $\int_a^b f'(t)e^{f(t)}\,dt = \left[e^{f(t)}\right]_a^b$, with $f(t) = -\left[0.005t^2+0.04t\right]$, so that $f'(t) = -(0.01t+0.04)$, we know that:

$$\int_0^5 -(0.01t+0.04)\exp\left(-\left[0.005t^2+0.04t\right]\right)dt = \left[\exp\left(-\left[0.005t^2+0.04t\right]\right)\right]_0^5$$

A redemption yield that is calculated without making any allowance for tax is called a *gross redemption yield* (GRY). If tax is incorporated in the calculation, this gives the *net redemption yield* (NRY).

We can determine the gross redemption yield for a fixed-interest stock by trial and error and then interpolation.

Question

A tax-exempt investor pays £10,500 for £10,000 nominal of a newly issued 5-year fixed-interest bond that is redeemable at par and pays coupons of 8% *pa* half-yearly in arrears.

Calculate the gross redemption yield obtained by the investor.

Solution

The gross redemption yield is the interest rate i that satisfies the equation of value:

$$10,500 = 800 a_{\overline{5}|}^{(2)} + 10,000 v^5$$

This is the bond referred to in earlier questions, and we've already seen that if the yield is 10% *pa* effective, the price is £9,315.85. Here the investor is paying more than £9,315.85, so the yield is lower than 10% *pa* effective. However, we can get a better first guess by considering the coupon rate and the redemption payment.

If the purchase price were £10,000, the redemption yield would be exactly 8% *pa* convertible half-yearly, because an investment of £10,000 would be receiving interest of £800 each year payable half-yearly, followed by a return of the initial investment. Since the investor pays more than £10,000, the redemption yield will be less than 8% *pa* convertible half-yearly. The effective redemption yield will be only slightly higher than the yield convertible half-yearly, so a good first guess is that the effective redemption yield will be lower than 8% *pa*.

At 7%: $RHS = 10,466.45$

At 6%: $RHS = 10,892.28$

We can approximate i by linearly interpolating using these two values:

$$i \approx 6\% + \frac{10,500 - 10,892.28}{10,466.45 - 10,892.28} \times (7\% - 6\%) = 6.9\%$$

So the GRY is approximately 6.9% *pa* effective.

1.2 No tax

Consider an n year fixed-interest security which pays coupons of D per annum, payable p thly in arrears and has redemption amount R.

The price of this bond, at an effective rate of interest i per annum, with no allowance for tax (*ie* i represents the gross yield) is:

$$P = Da_{\overline{n}|}^{(p)} + Rv^n \quad \text{at rate } i \text{ per annum} \tag{1.2}$$

We looked at an example of this earlier, as our tax-exempt investor received the full amount of the coupon and redemption payments.

Note: One could also work with a period of half a year. The corresponding equation of value would then be:

$$P = \frac{D}{2}a_{\overline{2n}|} + Rv^{2n} \quad \text{at rate } i' \text{ where } (1+i')^2 = 1+i$$

Question

A tax-exempt investor purchases £10,000 nominal of a newly issued 5-year fixed-interest bond that is redeemable at par and pays coupons of 8% *pa* half-yearly in arrears.

Calculate the price the investor should pay to obtain an effective annual yield of 10%.

Solution

We have already calculated the answer to this question by working in years. The price worked out to be £9,315.85. We will confirm this by working in half-years, using a half-yearly effective interest rate.

First we need to calculate the half-yearly effective rate:

$$1.1^{½} - 1 = 4.880885\%$$

So the price to be paid is:

$$P = 400a_{\overline{10}|} + 10{,}000v^{10} \quad @\,4.880885\%$$

$$= 400 \times 7.7666 + 10{,}000 \times 1.04880885^{-10}$$

$$= £9{,}315.85$$

Letting P denote the price for £100 nominal of the bond:

$$P = 2a_{\overline{4}|6\%} + 102v_{7\%}^5 = 2 \times 3.4651 + 102 \times 1.07^{-5} = £79.65$$

The gross redemption yield is the interest rate, i, that satisfies the equation of value:

$$79.65 = 2a_{\overline{5}|} + 100v^5$$

The gross redemption yield is a weighted average of the interest rates over the term of the bond, where the weights are the present values of the cashflows that occur at the different durations. The gross redemption yield is likely to be close to 7% here, as that is the spot rate associated with the duration of the largest cashflow (the redemption payment).

At 7%, the right-hand side gives £79.50, and at 6.5%, the right-hand side gives £81.30. Linearly interpolating, we find the gross redemption yield to be:

$$i \approx 6.5\% + \frac{79.65 - 81.30}{79.50 - 81.30} \times (7\% - 6.5\%) = 6.96\%$$

(ii) ***4% coupon rate***

In this case, the price for £100 nominal of the bond is:

$$P = 4a_{\overline{4}|6\%} + 104v_{7\%}^5 = 4 \times 3.4651 + 104 \times 1.07^{-5} = £88.01$$

The gross redemption yield is the interest rate, i, that satisfies the equation of value:

$$88.01 = 4a_{\overline{5}|} + 100v^5$$

At 7%, the right-hand side gives £87.70, and at 6.5% the right-hand side gives £89.61. Linearly interpolating, we find the gross redemption yield to be:

$$i \approx 6.5\% + \frac{88.01 - 89.61}{87.70 - 89.61} \times (7\% - 6.5\%) = 6.92\%$$

The gross redemption yield of the bond with the 4% coupon rate is lower than that for the bond with the 2% coupon rate. This is because the bond with the 4% coupon rate has higher cashflows at the earlier durations, so these gain more weighting in the calculation and, since the spot rates at the earlier durations are lower, the gross redemption yield is lower.

3.4 Par yields

We have already met the *yield to maturity* or the *redemption yield* for a fixed-interest investment. This is just the constant interest rate that satisfies the equation of value. For a zero-coupon bond, this is the same as the spot rate.

The *n-year par yield* represents the coupon per £1 nominal that would be payable on a bond with term *n* years, which would give the bond a current price under the current term structure of £1 per £1 nominal, assuming the bond is redeemed at par.

That is, if yc_n is the *n*-year par yield:

$$1 = (yc_n)(v_{y_1} + v_{y_2}^2 + v_{y_3}^3 + \cdots + v_{y_n}^n) + 1v_{y_n}^n$$

The par yields give an alternative measure of the relationship between the yield and term of investments.

The gross redemption yield of a bond depends on its term, price, coupon rate and redemption rate. Different bonds with the same term can have different gross redemption yields, so the gross redemption yield on its own does not provide a simple measure of how interest rates vary by term.

The par yield aims to provide us with that simple measure by considering a *notional bond* with a particular term, rather than specific bonds. This notional bond has a standardised structure in which the price is equal to the redemption payment, so there is no capital gain (or loss) and the only return is provided by the coupons. If the yield curve is flat (*ie* with all the spot/forward rates being equal to one 'market' rate of interest at all terms), then the par yield would be equal to that market interest rate, and would be the same for all terms. The extent to which the par yield *does* vary by term, therefore, reflects the way in which the underlying spot/forward rates of interest vary by term.

The difference between the par yield rate and the spot rate is called the 'coupon bias'.

The spot rate for a given term is the yield on a zero-coupon bond of that term, whilst the par yield for a given term is the yield on a notional coupon-paying bond of that term. The coupon bias is then the difference (or bias) in yields between these two types of bond as a result of the coupons paid.

Question

Calculate the 5-year par yield if the annual term structure of interest rates is:

$$(6\%, 6.25\%, 6.5\%, 6.75\%, 7\%, \ldots)$$

ie $y_1 = 6\%$, $y_2 = 6.25\%$, $y_3 = 6.5\%$, $y_4 = 6.75\%$ and $y_5 = 7\%$.

Solution

The 5-year par yield yc_5 is found from the equation:

$$yc_5(v_{y_1} + v_{y_2}^2 + \cdots + v_{y_5}^5) + v_{y_5}^5 = 1$$

Using the spot rates given, this becomes:

$$yc_5(1.06^{-1} + 1.0625^{-2} + 1.065^{-3} + 1.0675^{-4} + 1.07^{-5}) + 1.07^{-5} = 1$$

ie: $yc_5 \times 4.1401 + 0.71299 = 1 \quad \Rightarrow \quad yc_5 = 6.93\%$

Chapter 20 Solutions

20.1 ***Prospective calculation***

The prospective reserve is equal to the expected present value (EPV) at time 5 of the future benefit outgo, minus the EPV at time 5 of the future premium income.

The expected present value of the future benefits is:

$$500{,}000\, A^{1}_{35:\overline{5}|} = 500{,}000\left(A_{35} - \frac{D_{40}}{D_{35}} A_{40}\right)$$

$$= 500{,}000\left(0.19219 - \frac{2{,}052.96}{2{,}507.40} \times 0.23056\right)$$

$$= 500{,}000 \times 0.00341659$$

$$= 1{,}708.29$$

The EPV of the future premiums is:

$$330.05\, \ddot{a}_{35:\overline{5}|} = 330.05\left(\ddot{a}_{35} - \frac{D_{40}}{D_{35}} \ddot{a}_{40}\right)$$

$$= 330.05\left(21.003 - \frac{2{,}052.96}{2{,}507.40} \times 20.005\right)$$

$$= 330.05 \times 4.6237$$

$$= 1{,}526.05$$

Hence the prospective reserve is:

$$1{,}708.29 - 1{,}526.05 = £182 \text{ to the nearest } £1$$

Retrospective calculation

The retrospective reserve is equal to the EPV of the first 5 years' premiums minus the EPV of the first 5 years' benefit payments, all accumulated to time 5.

The EPV of the first 5 years' premiums is:

$$330.05\, \ddot{a}_{30:\overline{5}|} = 330.05\left(\ddot{a}_{30} - \frac{D_{35}}{D_{30}} \ddot{a}_{35}\right)$$

$$= 330.05\left(21.834 - \frac{2{,}507.40}{3{,}060.13} \times 21.003\right)$$

$$= 330.05 \times 4.6246$$

$$= 1{,}526.36$$

The EPV of the first 5 years' benefits is:

$$500{,}000\,A^{\,1}_{30:\overline{5}|} = 500{,}000\left(A_{30} - \frac{D_{35}}{D_{30}}A_{35}\right)$$

$$= 500{,}000\left(0.16023 - \frac{2{,}507.40}{3{,}060.13}\times 0.19219\right)$$

$$= 500{,}000 \times 0.00275394$$

$$= 1{,}376.97$$

Hence the retrospective reserve is:

$$(1{,}526.36 - 1{,}376.97)\times\frac{D_{30}}{D_{35}} = (1{,}526.36 - 1{,}376.97)\times\frac{3{,}060.13}{2{,}507.40} = £182$$

which is the same as the prospective reserve.

20.2 The reserve at the end of the fifth policy year is:

$${}_5V^{pro} = EPV \text{ future benefits} + EPV \text{ future expenses} - EPV \text{ future premiums}$$

Now, writing $b = 0.0192308$:

EPV future benefits $=$

$$50{,}000\times 1.03^5 \times\left[\frac{1}{1.06}\times {}_{0|}q_{40} + \frac{1+b}{1.06^2}\times {}_{1|}q_{40} + \cdots + \frac{(1+b)^{19}}{1.06^{20}}\times {}_{19|}q_{40}\right]$$

$$+50{,}000\times 1.03^5 \times \frac{(1+b)^{20}}{1.06^{20}}\times {}_{20}p_{40}$$

$$= \frac{50{,}000\times 1.03^5}{1+b}\times\left[\frac{1+b}{1.06}\times {}_{0|}q_{40} + \left(\frac{1+b}{1.06}\right)^2 \times {}_{1|}q_{40} + \cdots + \left(\frac{1+b}{1.06}\right)^{20}\times {}_{19|}q_{40}\right]$$

$$+50{,}000\times 1.03^5 \times 1.04^{-20}\times {}_{20}p_{40}$$

$$= 50{,}000\times 1.03^5 \times\left[\frac{A^{\,1\ @4\%}_{40:\overline{20}|}}{1+b} + \frac{D^{@4\%}_{60}}{D^{@4\%}_{40}}\right]$$

where:

$$\frac{D^{@4\%}_{60}}{D^{@4\%}_{40}} = \frac{882.85}{2{,}052.96} = 0.43004$$

$$A^{\,1\ @4\%}_{40:\overline{20}|} = A^{@4\%}_{40:\overline{20}|} - \frac{D^{@4\%}_{60}}{D^{@4\%}_{40}} = 0.46433 - 0.43004 = 0.03429$$

So, the expected present value of the future benefits is:

$$50{,}000 \times 1.03^5 \times \left[\frac{0.03429}{1.0192308} + 0.43004\right] = 26{,}876.78$$

Hence:

$$ {}_5V^{pro} = 26{,}876.78 + 350A^{@6\%}_{40:\overline{20}|} - (1-0.05)\times 1{,}500 \times \ddot{a}^{@6\%}_{40:\overline{20}|}$$

$$= 26{,}876.78 + 350 \times 0.32088 - 0.95 \times 1{,}500 \times 11.998$$

$$= £9{,}892$$

20.3 The gross premium retrospective reserve can be calculated as the expected present value at the outset of (premiums – benefits – expenses), accumulated with interest and allowing for survival to age 45. This gives:

$$\frac{(1+i)^5}{{}_5p_{40}} \times \left[1{,}700\ddot{a}_{40:\overline{5}|} - 20{,}000A^{\;1}_{40:\overline{5}|} - 425 - 72 \times \left(\ddot{a}_{40:\overline{5}|} - 1\right)\right]$$

20.4 The prospective reserve is the expected present value at time 1 of the future benefits and expenses:

$$ {}_1V^{pro} = 1.015 \times 3{,}000\, a_{61:\overline{9}|}$$

The retrospective reserve is the retrospective accumulation of the premium less benefits and expenses:

$$ {}_1V^{retro} = \frac{D_{60}}{D_{61}}\left(P - 1.015 \times 3{,}000 a_{60:\overline{1}|} - 200\right)$$

where P is the single premium.

The premium equation is:

$$P = 1.015 \times 3{,}000 a_{60:\overline{10}|} + 200$$

Splitting this at time 1:

$$P = 1.015 \times 3{,}000\left[a_{60:\overline{1}|} + \frac{D_{61}}{D_{60}} a_{61:\overline{9}|}\right] + 200$$

Rearranging:

$$P - 1.015 \times 3{,}000 a_{60:\overline{1}|} - 200 = 1.015 \times 3{,}000 \frac{D_{61}}{D_{60}} a_{61:\overline{9}|}$$

Accumulating to time 1:

$$\frac{D_{60}}{D_{61}}\left(P-1.015\times 3{,}000\,a_{\overline{60:1|}}-200\right)=1.015\times 3{,}000\,a_{\overline{61:9|}}$$

That is:

$${}_1V^{retro}={}_1V^{pro}$$

20.5 The policy starts when the policyholder is age 50. The 5th premium is paid on the policyholder's 54th birthday, when the remaining term will be 6 years.

If death occurs at age 54 last birthday, the benefit amount will be £14,000, which will increase by £1,000 each year. The maturity value is £25,000. So the net premium reserve is:

$${}_4V^{pro}=13{,}000A^{1}_{\overline{54:6|}}+1{,}000(IA)^{1}_{\overline{54:6|}}+25{,}000\frac{D_{60}}{D_{54}}-P\ddot{a}_{\overline{54:6|}}$$

where the net premium is given by:

$$P\ddot{a}_{\overline{50:10|}}=9{,}000A^{1}_{\overline{50:10|}}+1{,}000(IA)^{1}_{\overline{50:10|}}+25{,}000\frac{D_{60}}{D_{50}}$$

In order to specify the calculation of the reserve precisely, it is necessary to state how the premium is calculated. This is because, for a net premium reserve, the net premium is always calculated on the same basis as the reserve.

20.6 The gross premium retrospective reserve at the end of year 20 is:

$${}_{20}V^{retro}=\frac{D_{30}}{D_{50}}\left[250\ddot{a}_{\overline{30:20|}}-30{,}000\bar{A}^{1}_{\overline{30:20|}}-100-0.05\times 250\times\left(\ddot{a}_{\overline{30:20|}}-1\right)\right]$$

20.7 The reserve at time 0 is zero, and so in the first year the equation of equilibrium is:

$$(P-I)(1+i)=(1+c)Sq_x+{}_1V\,p_x$$

For subsequent years, *ie* $t=1,2,\ldots$:

$$({}_tV+P-kP)(1+i)=(1+c)Sq_{x+t}+{}_{t+1}V\,p_{x+t}$$

20.8 (i) The retrospective accumulation at the end of 5 years is:

$$200(1.075)^5\frac{l_{25}}{l_{30}}=200(1.075)^5\times\frac{98{,}797}{98{,}617}=£287.65$$

(ii) The retrospective accumulation at the end of 20 years is:

$$200(1.075)^{20}\frac{l_{25}}{l_{45}}=200(1.075)^{20}\times\frac{98{,}797}{97{,}315}=£862.51$$

Hence the mortality profit for the group of policies for 2019 is:

$$EDS - ADS = £199,587$$ [½]

[Total 5]

23.3 (i) ***Definitions***

The 'death strain at risk' for a policy for year $t+1$ (*ie* the year beginning at time t and ending at time $t+1$, $t = 0,1,2,\dots$) is the excess of the sum assured (*ie* the value at time $t+1$ of all benefits payable on death during year $t+1$) over the end-of-year reserve. [1]

The 'expected death strain' for a group of policies for year $t+1$, is the total death strain that would be incurred in respect of all policies in force at the start of year $t+1$ if deaths conformed to the numbers expected.

$$\text{EDS for year } t+1 = \sum_{\substack{\text{policies in force}\\ \text{at start of year}}} q(S - {}_{t+1}V)$$ [1]

The 'actual death strain' for a group of policies for year $t+1$ is the total death strain incurred in respect of all claims actually arising during year $t+1$.

$$\text{ADS for year } t+1 = \sum_{\text{claims during year}} (S - {}_{t+1}V)$$ [1]

[Total 3]

(ii) ***Mortality profit***

The premiums per unit sum assured for the three types of policies can be found as follows:

$$P_a \, \ddot{a}_{45:\overline{20|}} = A_{45:\overline{20|}} \Rightarrow P_a = 0.46998 / 13.780 = 0.03411$$ [1]

$$P_b \, \ddot{a}_{45:\overline{20|}} = A^{1}_{45:\overline{20|}}$$

where:

$$A^{1}_{45:\overline{20|}} = A_{45:\overline{20|}} - \frac{D_{65}}{D_{45}} = 0.46998 - \frac{689.23}{1,677.97} = 0.05923$$

$$\Rightarrow P_b = 0.05923 / 13.780 = 0.00430$$ [2]

$$P_c = P_a - P_b = 0.02981$$ [½]

The reserves at the end of the year per unit sum assured are:

$$ {}_{10}V_a = A_{55:\overline{10|}} - P_a\,\ddot{a}_{55:\overline{10|}} = 0.68388 - 0.03411 \times 8.219 = 0.4036 \qquad [1]$$

$$ {}_{10}V_b = A^{1}_{55:\overline{10|}} - P_b\ddot{a}_{55:\overline{10|}} = 0.06037 - 0.00430 \times 8.219 = 0.02505 \qquad [1]$$

$$ {}_{10}V_c = \frac{D_{65}}{D_{55}} - P_c\ddot{a}_{55:\overline{10|}} = 0.62351 - 0.02981 \times 8.219 = 0.3785 \qquad [1]$$

The total expected death strain is:

$$\begin{aligned} EDS &= EDS_a + EDS_b + EDS_c \\ &= q_{54}[600{,}000(1 - {}_{10}V_a) + 200{,}000(1 - {}_{10}V_b) + 80{,}000(0 - {}_{10}V_c)] \\ &= 0.003976[600{,}000(1 - 0.4036) + 200{,}000(1 - 0.02505) + 80{,}000(-0.3785)] \\ &= 2{,}078 \end{aligned} \qquad [2]$$

The total actual death strain is:

$$\begin{aligned} ADS &= ADS_a + ADS_b + ADS_c \\ &= 4{,}000(1 - 0.4036) + 2{,}000(1 - 0.02505) + 500(-0.3785) \\ &= 4{,}146 \end{aligned} \qquad [2]$$

So there is a profit of $2{,}078 - 4{,}146 = -2{,}069$, *ie* a loss of £2,069. [½]

[Total 11]

23.4 If the policyholder dies in the fourth policy year, the curtate duration will equal 3. So the sum assured payable immediately on death during this year is:

$$100{,}000 - 3 \times 10{,}000 = 70{,}000$$

In the death strain at risk, this needs accumulating to the end of the year. So, assuming deaths occur on average half way through the year, we need to multiply this by $1.04^{½}$. From this we need to deduct the payment made on survival at the end of the year (which is the annuity payment of 10,000) and the reserve at the end of the year. So the death strain at risk in the fourth policy year is:

$$DSAR = 70{,}000 \times 1.04^{½} - 10{,}000 - {}_4V \qquad [2]$$

where ${}_4V$ is the reserve at the end of year 4.

There are no future premiums or expenses, so the reserve is equal to the expected present value (EPV) at time 4 (when the policyholder is aged 59 exact) of the future death benefits and annuity benefits.

The death benefit in year 5 is 60,000; in year 6, it is 50,000, and so on, decreasing by 10,000 each year until by year 10 it has reduced to 10,000. We can calculate the EPV of these benefits by deducting an increasing term assurance with 10,000 *pa* increases from a level term assurance with a sum assured of 70,000. So, including the EPV of the remaining annuity payments, the reserve becomes:

$$_4V = 70{,}000\bar{A}^{\,1}_{59:\overline{6}|} - 10{,}000(I\bar{A})^{\,1}_{59:\overline{6}|} + 10{,}000\,a_{59:\overline{6}|}$$ [1]

To work out these factors, we will need:

$$\frac{D_{65}}{D_{59}} = \frac{689.23}{924.76} = 0.74531$$

Now:

$$\bar{A}^{\,1}_{59:\overline{6}|} \approx (1+i)^{½}\left(A_{59} - \frac{D_{65}}{D_{59}}A_{65}\right)$$

$$= 1.04^{½} \times \left(0.44258 - 0.74531 \times 0.52786\right)$$

$$= 0.050136$$ [1]

$$(I\bar{A})^{\,1}_{59:\overline{6}|} \approx (1+i)^{½}\left\{(IA)_{59} - \frac{D_{65}}{D_{59}}\left[(IA)_{65} + 6A_{65}\right]\right\}$$

$$= 1.04^{½} \times \left(8.42588 - 0.74531 \times \left[7.89442 + 6 \times 0.52786\right]\right)$$

$$= 0.185205$$ [1½]

$$a_{59:\overline{6}|} = \left(\ddot{a}_{59} - 1\right) - \frac{D_{65}}{D_{59}}\left(\ddot{a}_{65} - 1\right) = 13.493 - 0.74531 \times 11.276 = 5.089$$ [1]

So:

$$_4V = 70{,}000 \times 0.050136 - 10{,}000 \times 0.185205 + 10{,}000 \times 5.089 = 52{,}546.66$$ [½]

Hence:

$$DSAR = 70{,}000 \times 1.04^{½} - 10{,}000 - 52{,}546.66 = 8{,}839.61$$ [½]

8 of the initial 1,500 policyholders received two or fewer annuity payments, which means that 8 policyholders died during the first three policy years. So there were 1,492 policies still in force at the start of the fourth policy year, when the policyholders were all aged 58. [½]

So the expected death strain was:

$$EDS = 1{,}492 \times q_{58} \times DSAR = 1{,}492 \times 0.006352 \times 8{,}839.61 = 83{,}775$$ [1]

4 policyholders received exactly 3 annuity payments, which means they must have died during the fourth policy year. So the actual death strain was:

$$ADS = 4 \times 8,839.61 = 35,358$$ [½]

and the mortality profit was:

$$EDS - ADS = 83,775 - 35,358 = £48,416$$ [½]

[Total 10]

23.5 (i) ***Mortality profit***

The reserve required (per policy) at the end of the 8th year can be found from the equation of equilibrium:

$$1.04 \times \left({}_7V + P \right) = q_{57} \times 10,000 + p_{57} \times {}_8V$$ [1]

Inserting the values gives:

$$1.04 \times (12,951 + 1,591) = 0.00995 \times 10,000 + 0.99005 \times {}_8V$$

So:

$${}_8V = 15,024.18 / 0.99005 = 15,175.17$$ [1]

The expected death strain is:

$$200q_{57}(10,000 - {}_8V) = 1.99(10,000 - 15,175.17) = -10,298.59$$ [1]

The actual death strain is:

$$3(10,000 - {}_8V) = 3(10,000 - 15,175.17) = -15,525.52$$ [1]

So the mortality profit for the year is:

$$EDS - ADS = -10,298.59 - (-15,525.52) = £5,227$$ [1]

[Total 5]

(ii) ***Comment***

In this case the reserve exceeds the death benefit, so the company makes a profit when people die. More people than expected died, so the result is a mortality profit. [1]

23.6 *This question is Subject CT5, April 2016, Question 11.*

We need the mortality profit for calendar year 2014, at the start of which all male and female policyholders were aged exactly 67 and 62 respectively.

2.3 Incorporating non-unit reserves into the profit test

The assumptions (of interest and mortality – and indeed expenses) that we use for calculating the non-unit reserves will be the insurer's valuation (or reserving) basis that was referred to towards the end of Section 1. In other words, we calculate the non-unit reserves that zeroise the expected negative cashflows that arise *when using the reserving basis*.

These reserves so calculated will then be put into the company's pricing profit-test model, as also described in that section.

Question

In an earlier example, we looked at a five-year unit-linked contract which had in-force expected cashflows (before reserves) of $(-60.20, -20.50, -17.00, 50.13, 85.75)$. Assuming 5% *pa* interest and mortality probabilities:

$$q_{55+t} = 0.01 + 0.001t \quad \text{for } t = 0, 1, \ldots, 4$$

it was found that this policy required non-unit reserves of ${}_1V = 34.77$ and ${}_2V = 16.19$. These reserves resulted in a profit vector of $(-94.62, 0, 0, 50.13, 85.75)$, when calculated on the same basis.

Assume that the assumptions that were used for these calculations were more prudent than the insurer's current best estimate basis.

Without doing any calculations, explain how the profit vector produced would be expected to change, when we run the company's pricing profit-test model using best estimate assumptions and incorporating the above reserves.

Solution

The profit test basis will be less cautious than the reserving basis. This means we will have some or all of the following:

- lower expected expenses and expense inflation
- higher investment returns
- lower expected claim costs

than before. These will in turn lead to having:

- higher expected profits from expenses and expense inflation
- higher investment profits
- higher mortality profits

than before. As this will apply to every year of the projection, all the profits will be larger (*ie* become more positive or less negative) than they were when calculated using the reserving basis.

This means the expected profits in Years 2 and 3 in this example will now be *greater* than zero, not equal to zero. So, while our non-unit reserves will zeroise profits in certain years on the *reserving* basis, our *best estimates* of profits in those years will be *greater* than zero.

When answering profit-testing questions where non-unit reserves are required, these reserves should be calculated on the basis specified for these in the question.

2.4 Full unit-linked example profit test

The following example is quite long and complex, as it involves calculations on two bases (the reserving basis, and the profit test basis). This level of computation is beyond what would be expected in the paper-based exam, but is possible in the computer-based assessment. It would be good practice to try to match the figures obtained below in Excel.

Policy details

A life insurance company issues a 4-year unit-linked endowment assurance policy to a person currently aged exactly 50, that offers the following benefits:

- on death, the higher of the unit fund value or 50,000 is paid
- on surrender, the unit fund value less a surrender penalty is paid. The surrender penalty is 225 if surrender occurs at the end of the first year, 150 if it occurs in the second year, and 75 if it is in the third year
- on maturity, the unit fund value is paid.

The death and surrender benefits are payable at the end of the policy year of claim, based on unit fund values at the end of the year, after deduction of all charges.

Premiums of 3,000 are payable annually in advance throughout the term of the policy, ceasing immediately on earlier death or on surrender of the policy.

The contract has the following charging structure:

- an allocation rate in the first year of 80% of the first premium
- allocation rate in all other years equal to 104% of each premium
- bid-offer spread: 5%
- fund management charge: 0.75% of the unit fund value deducted at the end of each year.

Profit test experience basis

The company currently uses the following basis to profit test this policy:

- AM92 Select mortality.
- Surrenders in the first year occur at a rate of 15% of all policies in force at the end of the year. In the second year, the rate is 8%, and in the third year the rate is 3%.
- Unit growth rate: 5% per annum effective.
- Interest earned on non-unit cashflows: 2% per annum effective.

- Initial expenses: 300.
- A renewal expense of 40 is incurred at the start of each year except the first.
- Non-unit reserves are set up at the start of each year that will exactly zeroise any negative cashflows that are expected according to the company's current reserving basis (see below). Negative non-unit reserves are not permitted.
- Risk discount rate: 7% per annum effective.

Reserving basis

- AM92 Select mortality.
- Surrenders are ignored.
- Unit growth rate: 3% per annum effective.
- Interest earned on non-unit cashflows 1% per annum effective.
- Initial expenses: 300.
- A renewal expense of 50 is incurred at the start of each year except the first.

Question

(i) Calculate the required non-unit reserves per policy in force at the start of each year.

(ii) Calculate the profit margin for this policy.

Solution

(i) ***Non-unit reserves***

We need to work out the non-unit reserves required according to the assumptions set out in the reserving basis. The first step is to project the unit fund using the reserving basis assumptions.

Unit fund (reserving basis)

The unit fund projection using the reserving basis is shown in the table below:

Policy year	Cost of allocation (1)	Total unit fund at start of year (2)	End of year unit fund before FMC (3)	Fund management charge (FMC) (4)	End of year unit fund after FMC (5)
1	2,280.00	2,280.00	2,348.40	17.61	2,330.79
2	2,964.00	5,294.79	5,453.63	40.90	5,412.73
3	2,964.00	8,376.73	8,628.03	64.71	8,563.32
4	2,964.00	11,527.32	11,873.14	89.05	11,784.09

where:

$(1) = \{\text{premium}\} \times \{\text{allocation rate}\} \times \left(1 - \{\text{bid-offer spread}\}\right)$

$(2)_t = (5)_{t-1} + (1)_t$ for year $t = 2,3,4$

$(3) = (2) \times \left(1 + \{\text{unit growth rate}\}\right) = (2) \times 1.03$

$(4) = (3) \times \{\text{fund management charge rate}\} = (3) \times 0.0075$

$(5) = (3) - (4)$

Next we need to calculate the expected non-unit cashflows at the end of each year per policy in force at the start of the year (the expected non-unit cashflows).

Non-unit cashflows (reserving basis)

Policy year	Premium minus cost of allocation (6)	Expenses (7)	Non-unit interest (8)	FMC (9)	Mortality rate (10)	Expected death cost (11)
1	720.00	300.00	4.20	17.61	0.001971	93.96
2	36.00	50.00	–0.14	40.90	0.002732	121.81
3	36.00	50.00	–0.14	64.71	0.003152	130.61
4	36.00	50.00	–0.14	89.05	0.003539	135.25

Policy year	Expected non-unit cashflow at end of year (*NUCF*) (12)
1	347.86
2	–95.05
3	–80.04
4	–60.34

where:

$$(6) = \{\text{premium}\} - (1)$$

$$(8) = \left[(6)-(7)\right] \times \{\text{non-unit interest rate}\} = \left[(6)-(7)\right] \times 0.01$$

$$(11) = \max\left[\{\text{minimum death benefit}\} - (5), 0\right] \times (10)$$

$$(12) = (6) - (7) + (8) + (9) - (11)$$

Non-unit reserves

The non-unit reserves are now calculated as the amount of money that we need to hold at the start of each year, that will lead to expected profits of (not less than) zero at the end of the year. For this we continue to use the reserving basis assumptions as necessary.

As the most distant negative cashflow is in year 4, we require the non-unit reserve at the start of year 4 to be:

$${}_3V = \frac{-NUCF_4}{1 + \{\text{non-unit interest rate}\}} = \frac{60.34}{1.01} = 59.74$$

As the expected non-unit cashflows in years 2 and 3 are also negative, the non-unit reserves at the start of year t $(t = 2, 3)$ can be calculated recursively using:

$${}_{t-1}V = \frac{-NUCF_t + \{\text{probability of staying in force over } [t-1, t]\} \times {}_tV}{1 + \{\text{non-unit interest rate}\}}$$

$$= \frac{-NUCF_t + \left(1 - \{\text{mortality rate}\}\right) \times {}_tV}{1.01}$$

The non-unit reserve at the start of year 1 is assumed to be zero. This leads to the following reserves:

Policy year	Expected non-unit cashflow at end of year (*NUCF*) (12)	Probability of staying in force over the year (13)	Non-unit reserve at start of year (${}_{t-1}V$) (14)
1	347.86	0.998029	0
2	–95.05	0.997268	230.57
3	–80.04	0.996848	138.21
4	–60.34	–	59.74

(ii) ***Profit margin***

We now need to calculate the expected non-unit profits using the assumptions set out in the profit test experience basis. This means first calculating the projected unit fund values and expected non-unit cashflows in a similar way to part (i), but using the profit test basis. The resulting values are shown in the following tables.

Unit fund (profit test basis)

Policy year	Cost of allocation (15)	Total unit fund at start of year (16)	End of year unit fund before FMC (17)	Fund management charge (FMC) (18)	End of year unit fund after FMC (19)
1	2,280.00	2,280.00	2,394.00	17.96	2,376.05
2	2,964.00	5,340.05	5,607.05	42.05	5,564.99
3	2,964.00	8,528.99	8,955.44	67.17	8,888.28
4	2,964.00	11,852.28	12,444.89	93.34	12,351.56

Non-unit cashflows (profit test basis)

Policy year	Premium minus cost of allocation (20)	Expenses (21)	Non-unit interest (22)	FMC (23)	Mortality rate (24)	Expected death cost (25)
1	720.00	300.00	8.40	17.96	0.001971	93.87
2	36.00	40.00	–0.08	42.05	0.002732	121.40
3	36.00	40.00	–0.08	67.17	0.003152	129.58
4	36.00	40.00	–0.08	93.34	0.003539	133.24

Policy year	Survival probability (26)	Probability of surrender (27)	Expected surrender profit (28)	Expected non-unit cashflow (29)
1	0.998029	0.149704	33.68	386.17
2	0.997268	0.079781	11.97	–71.46
3	0.996848	0.029905	2.24	–64.26
4	–	–	–	–43.98

where:

$$(26) = 1 - (24)$$

$$(27) = (26) \times \{\text{surrender rate}\}$$

$$(28) = (27) \times \{\text{surrender penalty}\}$$

$$(29) = (20) - (21) + (22) + (23) - (25) + (28)$$

Profit vector

The next step is to calculate the expected profits at the end of each year, per policy in force at the start of the year (the profit vector). This means allowing for the impact of the non-unit reserves that we calculated in part (i).

Policy year	Expected non-unit cashflow (29)	Non-unit reserve at start of year (14)	Interest on reserve (30)	Probability of staying in force (31)	Expected cost of end year reserve (32)	Profit vector (33)
1	386.17	0	0	0.848325	195.60	190.57
2	–71.46	230.57	4.61	0.917487	126.80	36.93
3	–64.26	138.21	2.76	0.966943	57.77	18.95
4	–43.98	59.74	1.19	–	–	16.95

where:

$$(30) = (14) \times \{\text{non-unit interest rate}\} = (14) \times 0.02$$

$$(31) = 1 - \{\text{mortality probability}\} - \{\text{surrender probability}\} = (26) - (27)$$

$$(32)_t = (31)_t \times (14)_{t+1} \quad \text{for year } t = 1,2,3$$

$$(33) = (29) + (14) + (30) - (32)$$

If the profit test assumptions had been identical to the reserving basis, the profits in years 2-4 would have been exactly equal to zero. We expect positive profits here because:

- *the non-unit reserves will cover the level of negative cashflows expected on the* ***reserving*** *basis*
- *the profit test assumptions are less cautious than the reserving basis: for example, the unit growth rate is higher (producing higher expected fund management charges) and we are including the expected profits from surrender, producing higher (less negative) expected cashflows than before*

so the reserves held will be ***more*** *than enough to cover the expected level of negative cashflows on the* ***profit test*** *basis, leading to surplus cashflow (ie a profit) being expected each year.*

Net present value

We now need the net present value of the policy, which is calculated by multiplying each year's profit by the probability of the policy remaining in force from policy outset to the start of the year, and then discounting the resulting profit signature at the risk discount rate: summing the resulting values gives the net present value.

The net present value (NPV) is obtained using the figures calculated in the following table:

Policy year	Profit vector (33)	Probability of staying in force to start of year (34)	Profit signature (35)	Discount (36)	Expected present value (37)
1	190.57	1	190.57	0.934579	178.10
2	36.93	0.848325	31.33	0.873439	27.36
3	18.95	0.778326	14.75	0.816298	12.04
4	16.95	0.752597	12.76	0.762895	9.73

where:

$$(34)_t = (34)_{t-1} \times (31)_{t-1} \quad \text{for year } t = 2,3,4$$

$$(35) = (33) \times (34)$$

$$(36)_t = \left(1 + \{\text{risk discount rate}\}\right)^{-t} = 1.07^{-t} \quad \text{for year } t = 1,2,3,4$$

$$(37) = (35) \times (36)$$

The NPV is then the sum of the values in Column 37, which is 227.24.

Profit margin

The profit margin is equal to the NPV divided by the expected present value of the premiums using the profit test basis discounted at the risk discount rate.

The expected present value of the premiums is:

$$3{,}000 \times \left(1 + \frac{0.848325}{1.07} + \frac{0.778326}{1.07^2} + \frac{0.752597}{1.07^3}\right) = 9{,}260.97$$

Hence the profit margin is:

$$\frac{227.24}{9{,}260.97} = 2.45\%$$

3 Calculating reserves for conventional contracts using a profit test

In the previous section we saw how to determine non-unit reserves for unit-linked products by working backwards from the last negative cashflow. We can apply exactly the same methodology to conventional contracts.

A profit test can also be used to determine the reserves for a conventional (*ie* non unit-linked) policy. We illustrate the procedures by using a without-profit endowment assurance with a term of n years, a sum assured of S payable at the end of the year of death or on survival to the end of the term, and a surrender value payable at the end of year t of U_t, which is secured by a level annual premium of P.

A basis is required for the projection of the cashflows and for calculating the required reserves. This will consist of an interest rate i, dependent probabilities of death and surrender $(aq)^d_{x+t}$ and $(aq)^s_{x+t}$, dependent probabilities of remaining in force ${}_t(ap)_x$, and expenses per policy in force at time t of e_t.

$(CF)_t$, the expected cashflow at time t per policy in force at time $t-1$, ignoring reserves, is:

$$(CF)_t = \left(P - e_{t-1}\right)(1+i) - S\,(aq)^d_{x+t-1} - U_t\,(aq)^s_{x+t-1} \qquad t = 1, 2, 3, \ldots, n-1$$

$$(CF)_n = \left(P - e_{n-1}\right)(1+i) - S$$

assuming there is no surrender value paid at the end of the last policy year.

Technically it is not possible to surrender an annual premium endowment assurance in the last policy year, because all the premiums will have been paid by then and the policy will simply terminate as a maturity, receiving the full sum assured in benefit payment.

These cashflows will usually be positive for earlier years of a contract and negative during the later years. For example a five-year endowment assurance with a sum assured of 1,000 might have cashflows:

t	1	2	3	4	5
$(CF)_t$	156.39	187.41	186.33	185.14	–803.17

If the contract is to be self-funding, then reserves must be established using the earlier positive cashflows. These reserves should be sufficient to pay the later expected negative cashflows. This requirement is exactly analogous to the need to establish reserves in the non-unit fund for a unit-linked contract. Reserves can be established for conventional assurances using the same procedures as those used to establish non-unit reserves.

Let $(CF)_t$ denote the expected cashflow at the end of Year t, and let $(PRO)_t$ denote the t th entry in the profit vector. Calculations begin at the longest policy duration, m, at which there is a negative cashflow.