Subject CM1

CMP Upgrade 2020/21

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus objectives, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2020 CMP to make it suitable for study for the 2021 exams. It includes replacement pages and additional pages where appropriate. Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our 2021 *Student Brochure* for more details.

This CMP Upgrade contains:

- all significant changes to the Syllabus objectives and Core Reading
- additional changes to the ActEd Course Notes and Assignments that will make them suitable for study for the 2021 exams.

1 Changes to the Syllabus

There are no significant changes to the CM1 Syllabus.

2 Changes to the Core Reading and ActEd text

This section contains all the ***non-trivial*** changes to the Core Reading.

Changes to course parts

The Course Notes are split into five parts, which correspond with the content of the X Assignments and Tutorials (five days). The split of chapters in each part has changed in the 2021 Course Notes as follows:

- Part 1 is now Chapter 1 to Chapter 9.
- Part 2 is now Chapter 10 to Chapter 13.
- Part 3 is now Chapter 14 to Chapter 18.
- Parts 4 and 5 are unchanged.

Chapter 3

Page 19

The fifth paragraph on this page should refer to the effective rate of discount, not the effective rate of interest.

Chapter 6

Page 13

The solution to part (ii) of the question on this page should refer to the value of the rental payments on 1 January 2018, not 1 January 2023.

Chapter 7

Page 14

About halfway down the page, the $s_{\overline{n}|}^{(p)}$ term on the left hand side of the expression just after the words "Again, using the formula for $s_{\overline{n}|}$ gives:" should have double dots over it, *ie*:

$$\ddot{s}_{\overline{n}|}^{(p)} = \frac{(1+i)^n - 1}{d^{(p)}}$$

Chapter 8

Page 7

The y-axes of the graphs at the bottom of this page should be labelled “Rate of payment” rather than “Amount”.

Chapter 18

Page 10

In the solution on this page, the first line of calculations after the word ‘So:’ should read as follows:

$$(I\bar{A})_{50:\overline{10}|} \approx (1+i)^{1/2}\,(IA)^{\;1}_{50:\overline{10}|} + 10A^{\;\;1}_{50:\overline{10}|}$$

ie there should be no ‘bar’ on the increasing assurance after the $\approx$ sign.

3 Changes to the X Assignments

Overall

Due to the updated split of the Course Notes, there are some significant changes to Assignments X2 and X3. There is also a minor change to Assignment X1, which is unrelated to the updated course split. Details of these changes are given below.

Assignment X1

Question X1.7 has been re-worded to make it clear that the annuity should always have a ten-year payment period, regardless of when payments start. There are no changes to the solution.

Question

An investor, who has a sum of £10,000 to invest, wishes to purchase an annuity certain which makes payments over a ten-year period. Calculate the amount of the payments that can be provided if the annuity takes each of the following forms (assuming interest of 8% pa effective):

(i) a level annuity payable monthly in arrears [2]

(ii) a level annuity due payable half-yearly, commencing in 2 years' time. [2]
[Total 4]

Assignment X2

Questions X2.2, X2.4 and X2.9 part (i) (a) have been removed. Questions X3.1 and X3.12 have been brought in from Assignment X3 as these cover material that's now in Part 2 of the course. The table below maps 2020 Assignment X2 and X3 questions to the 2021 Assignment X2:

2020 question number	***2021 question number***
X3.1	X2.1
X2.1	X2.2
X2.2	*N/A – removed*
X2.3	X2.3
X2.4	*N/A – removed*
X2.5	X2.4
X3.12	X2.5
X2.6	X2.6
X2.7	X2.7
X2.8	X2.8
X2.9 excluding (i) (a)	X2.9

Some minor changes have been made to some of the questions for 2021, which are detailed below.

X2.3 (question number unchanged from 2020)

The course reference in the solution to X2.3 has been corrected to say:

Optional redemption dates are covered in Chapter 12, Section 1.5.

X2.7 (question number unchanged from 2020)

The wording of part (ii) of the question has been updated for clarity, as follows:

(ii) Calculate the total amount of money owed by the borrower immediately after the third instalment has been paid. [2]

X2.9 (question number unchanged from 2020)

Question X2.9 is now worth 19 marks in total, due to the removal of (i) (a). Part (i) of this question now reads:

(i) Explain what is meant by the discounted payback period from an investment project. [2]

The relevant part of the solution has also been removed.

Assignment X3

Questions X3.1, X3.4, X3.12 and X3.13 have been removed from Assignment X3. Questions X3.1 and X3.12 have been moved to Assignment X2, as detailed above, whereas X3.4 and X3.13 have been removed entirely. Three new questions have been added to Assignment X3 replace the four questions that have been removed.

The table below maps 2020 Assignment X3 questions to the 2021 Assignment X3:

2020 question number	***2021 question number***
X3.1	*X2.1*
X3.2	X3.1
X3.3	X3.2
X3.4	*N/A – removed*
X3.5	X3.3
N/A – see below	*New question* X3.4
X3.6	X3.5
X3.7	X3.6
N/A – see below	*New question* X3.7
X3.8	X3.8
X3.9	X3.9
X3.10	X3.10
X3.11	X3.11
X3.12	*X2.5*
X3.13	*N/A – removed*
X3.14	X3.12
N/A – see below	*New question* X3.13

Changes to existing questions are detailed below, followed by the new questions for 2021.

X3.3 (was X3.5 in 2020)

The solution to X3.3 (X3.5 for 2020) has been corrected. In the first line of the solution, the age on the deferred annuity should be 40 select, *ie*:

$$\text{EPV annuity benefit} = 15{,}000\ {}_{25|}\bar{a}_{[40]} = 15{,}000\frac{D_{65}}{D_{[40]}}\bar{a}_{65}$$

X3.10 (question number unchanged from 2020)

The following wording has been added to the end of the solution:

The final numerical answers above, particularly the variance, are very sensitive to rounding.

X3.4 (new question for 2021)

Question

Calculate the exact value of $\bar{A}^{\,1}_{70:\overline{1}|}$ assuming the force of mortality is constant between consecutive integer ages. [5]

Basis Mortality: ELT15 (Males)

Interest: 7.5% *pa* effective

Solution

Term assurance contracts are introduced in Chapter 15, Section 2. Calculating expected present values under a constant force of mortality is covered in Chapter 17, Section 5.

The expected present value of the term assurance can be written as:

$$\bar{A}^{\,1}_{70:\overline{1}|} = \int_0^1 v^t \, {}_tp_{70} \, \mu_{70+t} \, dt$$ [½]

We're told to assume that the force of mortality between consecutive integer ages is constant, so the force of mortality, μ, that applies between ages 70 and 71 exact can be calculated as:

$$\mu = -\ln\left(p_{70}\right) = -\ln\left(1-q_{70}\right) = -\ln\left(1-0.03930\right) = 0.040093$$ [1]

Since μ is constant, we also have for $0 \le t < 1$:

$${}_tp_{70} = e^{-\mu t} = e^{-0.040093t}$$ [½]

Substituting these values into the integral above gives:

$$\bar{A}^{\,1}_{70:\overline{1}|} = \int_0^1 1.075^{-t} e^{-0.040093t} \, 0.040093 \, dt$$

$$= 0.040093 \int_0^1 e^{-\left(0.040093+\ln 1.075\right)t} \, dt$$ [1]

Integrating:

$$\bar{A}^{\,1}_{70:\overline{1}|} = \frac{0.040093}{-\left(0.040093+\ln 1.075\right)} \left[e^{-\left(0.040093+\ln 1.075\right)t} \right]_0^1$$

$$= 0.356656\left(1-e^{-0.112414}\right)$$

$$= 0.037922$$ [2]

So the expected present value of the term assurance is around 0.0379.

[Total 5]

X3.7 (new question for 2021)

Question

A population is subject to the force of mortality $\mu_x = 0.03e^{0.015x-1.8}$.

Calculate the probability that a life now aged 35 exact:

(i) survives to age 65 exact [3]

(ii) dies between ages 65 exact and 80 exact. [3]
[Total 6]

Solution

Calculating probabilities without using a life table is covered in Chapter 14, Section 4.

(i) ***Survival to age 65 exact***

The probability can be written as:

$$ {}_{30}p_{35} = \exp\left(-\int_0^{30} \mu_{35+s}\, ds\right) = \exp\left(-\int_0^{30} 0.03e^{0.015(35+s)-1.8} ds\right)$$

$$= \exp\left(-\int_0^{30} 0.03e^{0.015s-1.275} ds\right) \qquad [1]$$

Evaluating the integral gives:

$$\int_0^{30} 0.03e^{0.015s-1.275} ds = 0.03e^{-1.275}\int_0^{30} e^{0.015s} ds = 0.03e^{-1.275}\left[\frac{e^{0.015s}}{0.015}\right]_0^{30}$$

$$= \frac{0.03e^{-1.275}}{0.015}\left(e^{0.015\times 30} - 1\right) = 0.31761 \qquad [1½]$$

So we have:

$$ {}_{30}p_{35} = \exp[-0.31761] = 0.72789 \qquad [½]$$

Alternatively, the probability can be written as:

$$ {}_{30}p_{35} = \exp\left(-\int_{35}^{65} \mu_s\, ds\right) = \exp\left(-\int_{35}^{65} 0.03e^{0.015s-1.8} ds\right) \qquad [1]$$

Evaluating the integral then gives:

$$\int_{35}^{65} 0.03e^{0.015s-1.8}ds = 0.03e^{-1.8}\int_{35}^{65} e^{0.015s}ds = 0.03e^{-1.8}\left[\frac{e^{0.015s}}{0.015}\right]_{35}^{65}$$

$$= \frac{0.03e^{-1.8}}{0.015}\left(e^{0.015\times 65} - e^{0.015\times 35}\right) = 0.31761 \qquad [1½]$$

The probability, ${}_{30}p_{35}$, can then be calculated as before. *[½]*

The other survival probabilities calculated in part (ii) below can also be calculated using this alternative approach.

[Total 3]

(ii) ***Death between age 65 and 80***

The probability of death between ages 65 and 80 can be calculated as follows:

$${}_{30}p_{35}\ {}_{15}q_{65} = {}_{30}p_{35}\left(1 - {}_{15}p_{65}\right) \qquad [½]$$

We've already worked out ${}_{30}p_{35}$ in part (i), so we just need to calculate ${}_{15}p_{65}$. Integrating the function for the force of mortality first:

$$\int_{0}^{15} 0.03e^{0.015(65+s)-1.8}ds = 0.03e^{-0.825}\int_{0}^{15} e^{0.015s}ds = 0.03e^{-0.825}\left[\frac{e^{0.015s}}{0.015}\right]_{0}^{15}$$

$$= \frac{0.03e^{-0.825}}{0.015}\left(e^{0.015\times 15} - 1\right) = 0.22115 \qquad [1½]$$

Using this to calculate ${}_{15}p_{65}$:

$${}_{15}p_{65} = \exp\left[-0.22115\right] = 0.80159 \qquad [½]$$

We can now calculate the required probability:

$${}_{30}p_{35}\left(1 - {}_{15}p_{65}\right) = 0.72789 \times \left(1 - 0.80159\right) = 0.14442 \qquad [½]$$

Alternatively, we could calculate the required probability using the relationship:

$${}_{30}p_{35}\ {}_{15}q_{65} = {}_{30}p_{35}\left(1 - {}_{15}p_{65}\right) = {}_{30}p_{35} - {}_{45}p_{35} \qquad [½]$$

Integrating the function for the force of mortality first gives:

$$\int_0^{45} 0.03e^{0.015(35+s)-1.8}ds = 0.03e^{-1.275}\int_0^{45} e^{0.015s}ds = 0.03e^{-1.275}\left[\frac{e^{0.015s}}{0.015}\right]_0^{45}$$ *[1½]*

$$= \frac{0.03e^{-1.275}}{0.015}\left(e^{0.015\times 45} - 1\right) = 0.53876$$

Now using this to calculate $_{45}p_{35}$*:*

$$_{45}p_{35} = \exp\left[-0.53876\right] = 0.58347$$ *[½]*

Finally, we have:

$$_{30}p_{35} - {}_{45}p_{35} = 0.72789 - 0.58347 = 0.14442$$ *[½]*

[Total 3]

X3.13 (new question for 2021)

Question

A pension fund provides its members with benefits on retirement at exact age 65. Members can choose to receive their benefits in one of three forms:

A A lump sum of £62,500 immediately on retirement, followed by a flat pension of £21,500, which is guaranteed for five years. Pension payments cease on the death of the member or at the end of the guarantee period, whichever is later.

B A lump sum of £100,000 paid on either the member's 80th birthday or on death, if earlier, plus a pension that starts at £20,000 and increases by 2.5% *pa* simple, payable until death.

C A pension that starts at £20,000 and increases in line with inflation, payable until the member dies, plus a lump sum payable on death of five times the most recent pension instalment.

Pensions are paid annually in advance. Increases are applied on the anniversary of the member's retirement (*ie* the first increase is applied at exact age 66). Lump sum death benefits are payable at the end of the year of death.

(i) Calculate the expected present values of the three benefit options. [9]

(ii) Discuss how a member might decide which benefit is best for them. You may assume that the member knows the expected present value of each option. [4]

Basis: Mortality: AM92 Select

Interest: 6% *pa* effective

Inflation: 1.92308% *pa* compound

[Total 13]

Solution

Annuities with guarantee periods are introduced in Chapter 16, Section 8. Annuities increasing by constant amounts are covered in Chapter 18, Section 3. Annuities increasing at compound rates are covered in Chapter 18, Section 2.

(i) ***Expected present values of the three benefit options***

A *Lump sum of £62,500 at retirement plus a pension of £21,500 guaranteed for five years*

The expected present value of the benefits under Option A is:

$$62{,}500+21{,}500\ddot{a}_{\overline{[65]:\overline{5|}}}=62{,}500+21{,}500\left(\ddot{a}_{\overline{5|}}+v^5\ {}_5p_{[65]}\ \ddot{a}_{70}\right)$$

$$=62{,}500+21{,}500\left(\frac{1-1.06^{-5}}{0.06/1.06}+1.06^{-5}\times\frac{8{,}054.0544}{8{,}772.7359}\times 9.140\right)$$

$$=62{,}500+21{,}500\left(4.46511+6.27042\right)$$

$$=£293{,}313$$ [3]

B *Pension starting at £20,000 and increasing by 2.5% pa simple, plus a lump sum of £100,000 at age 80 or earlier death*

The expected present value of the pension under Option B is:

$$19{,}500\ddot{a}_{[65]}+500\left(I\ddot{a}\right)_{[65]}=19{,}500\times 10.621+500\times 89.861$$

$$=252{,}040$$ [1]

The expected present value of the lump sum under Option B is:

$$100{,}000A_{[65]:\overline{15|}}=100{,}000\left(A_{[65]}-v^{15}\ {}_{15}p_{[65]}\ A_{80}+v^{15}\ {}_{15}p_{[65]}\right)$$

$$=100{,}000\left(0.39883-1.06^{-15}\ \frac{5{,}266.4604}{8{,}772.7359}\left(0.64501-1\right)\right)$$

$$=100{,}000\times 0.48775$$

$$=48{,}775$$ [1½]

The total expected present value of Option B is therefore:

$$252{,}040+48{,}775=£300{,}815$$ [½]

C *Pension starting at £20,000 and increasing by inflation (assumed to be 1.92308% pa compound) plus lump sum on death*

The expected present value of the pension benefit under Option C is:

$$20{,}000+20{,}000\times 1.0192308\times v\ p_{[65]}+20{,}000\times 1.0192308^2\times v^2\ {}_2p_{[65]}+\ldots$$

If we call the expected annual compound increase b, we can rewrite the above expression as:

$$20,000\left(1+(1+b)v\,p_{[65]}+(1+b)^2 v^2\,{}_2p_{[65]}+\dots\right)$$ [½]

Now, making the substitution that:

$$(1+b)v=\frac{1.0192308}{1.06}=\frac{1}{1.04}=V$$

we can rewrite the pension benefit as:

$$20,000\left(1+V\,p_{[65]}+V^2{}_2p_{[65]}+\dots\right)=20,000\ddot{a}_{[65]}^{@4\%}$$ [½]

Evaluating this using the *Tables*:

$$20,000\ddot{a}_{[65]}^{@4\%}=20,000\times 12.337$$
$$=246,740$$ [½]

The expected present value of the lump sum payable on death is:

$$100,000v\,q_{[65]}+100,000(1+b)v^2\,{}_{1|}q_{[65]}+100,000\times(1+b)^2v^3\,{}_{2|}q_{[65]}+\dots$$
$$=\frac{100,000}{(1+b)}\left((1+b)vq_{[65]}+(1+b)^2v^2\,{}_{1|}q_{[65]}+(1+b)^3v^3\,{}_{2|}q_{[65]}+\dots\right)$$ [½]

Using the substitution $(1+b)v=V$, as for the pension:

$$\frac{100,000}{(1+b)}\left(V\,q_{[65]}+V^2{}_{1|}q_{[65]}+V^3\,{}_{2|}q_{[65]}+\dots\right)$$
$$=\frac{100,000}{(1+b)}A_{[65]}^{@4\%}=\frac{100,000}{1.0192308}\times 0.52550=51,558$$ [½]

The total EPV of benefits under the second option is therefore:

$$246,740+51,558=£298,298$$ [½]

[Total 9]

(ii) ***How a member might choose their benefit***

While Option B has the highest expected present value, all three sets of benefits have very similar expected present values. [½]

This means that the expected present value alone is unlikely to form the basis for a member's decision. [½]

Each benefit structure contains a large lump sum, but the timing of these differ. Members may choose the benefit structure that provides them with a lump sum benefit when they most need it. [½]

For example, if a member wants to provide for a spouse or dependant after death, the third option may be more appropriate… [½]

…whereas a member who wants to pay off their mortgage on retirement might prefer the first option. [½]

The level of pension income is likely to inform decision-making as well. An increasing pension might be more desirable to some members than a flat one. [½]

For example, someone who believes their living expenses will increase broadly in line with inflation over time may prefer Option C as the pension is inflation-linked. [½]

While Option B provides an increasing income, the increases are fixed rather than being inflation-linked and are simple rather than compound. This means the member bears some risk that their income does not keep pace with inflation (particularly over the longer term). [½]

However, the member bears the highest risk of receiving a decreasing real income under Option A, since the pension does not increase and any (positive) inflation will erode the spending power of their income. [½]

A member who is in poorer than average health may choose Option A in order to benefit from the guarantee period and the higher income earlier on, *ie* to maximise their wealth in the short-term. [½]

A member who is more concerned about maximising their wealth in the medium-term may choose Option B… [½]

…and a member who is most concerned about what happens if they live a long time after retirement may choose Option C. [½]

Markers: give credit for any sensible point or example.

[Maximum 4]

4 Changes to the Y Assignments

There have been no changes to the Y Assignments.

5 Other tuition services

In addition to the CMP you might find the following services helpful with your study.

5.1 Study material

We also offer the following study material in Subject CM1:

- Flashcards
- Revision Notes
- ASET (ActEd Solutions with Exam Technique) and Mini-ASET
- Mock Exam and AMP (Additional Mock Pack).

For further details on ActEd's study materials, please refer to the 2021 *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

5.2 Tutorials

We offer the following (face-to-face and/or online) tutorials in Subject CM1:

- Regular Tutorials (five days)
- Block Tutorials (five days)
- A Preparation Day for the computer-based exam
- Six-day Bundles in both Regular and Block format, covering the five days for the Paper A exam, plus the Preparation Day for the computer-based exam.

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **www.ActEd.co.uk**.

5.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to the 2021 *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

5.4 Feedback on the study material

ActEd is always pleased to get feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course please send them by email to **CM1@bpp.com**.

