

# *Subject A213*

## 2021 Study Guide

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***Please note that Subject A213 is based on the Chapters 14-27 of the Institute and Faculty of Actuaries' (IFoA) Subject CM1. Therefore the majority of your study materials will refer to CM1 and not A213.***

***The chapters have not been renumbered and so the first chapter will be Chapter 14.***

## 0 The course structure

There are three parts to the Subject A213 course. The parts are broken down into chapters.

The following table shows how the parts, the chapters and the syllabus items relate to each other. We have also given you a broad indication of the length of each chapter. This table should help you plan your progress across the study session.

<i>Part</i>	<i>Chapter</i>	<i>Title</i>	<i>Syllabus objectives*</i>
<b>1</b>	14	The life table	4.1, 4.2
	15	Life assurance contracts	4.1, 4.2
	16	Life annuity contracts	4.1, 4.2
	17	Evaluation of assurances and annuities	4.2
	18	Variable benefits and conventional with-profits policies	4.1, 4.2
<b>2</b>	19	Gross premiums	6.1, 6.2
	20	Gross premium reserves	4.2, 6.2
	21	Joint life and last survivor functions	5.1
	22	Contingent and reversionary benefits	5.1
<b>3</b>	23	Mortality profit	6.3
	24	Competing risks	5.2, 5.3
	25	Unit-linked and accumulating with-profits contracts	4.1
	26	Profit testing	6.4
	27	Reserving aspects of profit testing	6.4, 6.5

The numbering of syllabus objectives is based on the IFoA Subject CM1's syllabus, rather than the syllabus published on the ASSA's website.

# 1 Before you start

When studying for the UK actuarial exams, you will need:

- a copy of the Formulae and Tables for Examinations of the Faculty of Actuaries and the Institute of Actuaries, 2nd Edition (2002) – these are often referred to as simply the Yellow Tables or the Tables
- a ‘permitted’ scientific calculator – you will find the list of permitted calculators on the profession’s website. Please check the list carefully, since it is reviewed each year.

These are both available from the Institute and Faculty of Actuaries’ eShop. Please visit [www.actuaries.org.uk](http://www.actuaries.org.uk).

## 2 Core study material

This section explains the role of the Syllabus, Core Reading and supplementary ActEd text. It also gives guidance on how to use these materials most effectively in order to pass the exam.

Some of the information below is also contained in the introduction to the Core Reading produced by the Institute and Faculty of Actuaries.

### **Syllabus**

The relevant individual Syllabus Objectives are included at the start of each course chapter and a complete copy of the Syllabus is included in this Study Guide. We recommend that you use the Syllabus as an important part of your study.

### **Core Reading**

The Core Reading has been produced by the Institute and Faculty of Actuaries. The purpose of the Core Reading is to ensure that tutors, students and examiners understand the requirements of the syllabus for the qualification examinations for Fellowship of the Institute and Faculty of Actuaries.

The Core Reading supports coverage of the syllabus in helping to ensure that both depth and breadth are re-enforced. It is therefore important that students have a good understanding of the concepts covered by the Core Reading.

The examinations require students to demonstrate their understanding of the concepts given in the syllabus and described in the Core Reading; this will be based on the legislation, professional guidance *etc* that are in force when the Core Reading is published, *ie* on 31 May in the year preceding the examinations.

Therefore the exams in April and September 2021 will be based on the Syllabus and Core Reading as at 31 May 2020. We recommend that you always use the up-to-date Core Reading to prepare for the exams.

Examiners will have this Core Reading when setting the papers. In preparing for examinations, students are advised to work through past examination questions and may find additional tuition helpful. The Core Reading will be updated each year to reflect changes in the syllabus and current practice, and in the interest of clarity.

### **Accreditation**

The Institute and Faculty of Actuaries would like to thank the numerous people who have helped in the development of the material contained in this Core Reading.

## ActEd text

Core Reading deals with each syllabus objective and covers what is needed to pass the exam. However, the tuition material that has been written by ActEd enhances it by giving examples and further explanation of key points. Here is an excerpt from some ActEd Course Notes to show you how to identify Core Reading and the ActEd material. **Core Reading is shown in this bold font.**

Note that in the example given above, the index *will* fall if the actual share price goes below the theoretical ex-rights share price. Again, this is consistent with what would happen to an underlying portfolio.

After allowing for chain-linking, **the formula for the investment index then becomes:**

$$I(t) = \frac{\sum_i N_{i,t} P_{i,t}}{B(t)}$$

**where  $N_{i,t}$  is the number of shares issued for the  $i$ th constituent at time  $t$ ;**

**$B(t)$  is the base value, or divisor, at time  $t$ .**

## Paper B Online Resources (PBOR)

The Paper B Online Resources (PBOR) will help you prepare for the computer-based paper. Delivered through a virtual learning environment (VLE), you will have access to worked examples and practice questions. PBOR also includes a Y Assignment, which is an exam-style assessment.

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*Legal action will be taken if these terms are infringed. In addition, we may seek to take disciplinary action through your Profession or through your employer.*

These conditions remain in force after you have finished using the course.

## 3 Skills

### Technical skills

The Core Reading and exam papers for these subjects tend to be very technical. The exams themselves have many calculation and manipulation questions. The emphasis in the exam will therefore be on *understanding* the mathematical techniques and applying them to various, frequently unfamiliar, situations. It is important to have a feel for what the numerical answer should be by having a deep understanding of the material and by doing reasonableness checks.

As a high level of pure mathematics and statistics is generally required for the Core Principles subjects, it is important that your mathematical skills are extremely good. If you are a little rusty you may wish to consider purchasing additional material to help you get up to speed. The course 'Pure Maths and Statistics for Actuarial Studies' is available from ActEd and it covers the mathematical techniques that are required for the Core Principles subjects, some of which are beyond A-Level (or equivalent) standard. You do not need to work through the whole course in order – you can just refer to it when you need help on a particular topic. An initial assessment to test your mathematical skills and further details regarding the course can be found on our website at [www.ActEd.co.uk](http://www.ActEd.co.uk).

### Study skills

#### *Overall study plan*

We suggest that you develop a realistic study plan, building in time for relaxation and allowing some time for contingencies. Be aware of busy times at work, when you may not be able to take as much study leave as you would like. Once you have set your plan, be determined to stick to it. You don't have to be too prescriptive at this stage about what precisely you do on each study day. The main thing is to be clear that you will cover all the important activities in an appropriate manner and leave plenty of time for revision and question practice.

Aim to manage your study so as to allow plenty of time for the concepts you meet in these courses to 'bed down' in your mind. Most successful students will probably aim to complete the courses at least a month before the exam, thereby leaving a sufficient amount of time for revision. By finishing the courses as quickly as possible, you will have a much clearer view of the big picture. It will also allow you to structure your revision so that you can concentrate on the important and difficult areas.

You can also try looking at our discussion forum on the internet, which can be accessed at [www.ActEd.co.uk/forums](http://www.ActEd.co.uk/forums) (or use the link from our home page at [www.ActEd.co.uk](http://www.ActEd.co.uk)). There are some good suggestions from students on how to study.

#### *Study sessions*

Only do activities that will increase your chance of passing. Try to avoid including activities for the sake of it and don't spend time reviewing material that you already understand. You will only improve your chances of passing the exam by getting on top of the material that you currently find difficult.

Ideally, each study session should have a specific purpose and be based on a specific task, eg *'Finish reading Chapter 3 and attempt Practice Questions 3.4, 3.7 and 3.12'*, as opposed to a specific amount of time, eg *'Three hours studying the material in Chapter 3'*.

Try to study somewhere quiet and free from distractions (eg a library or a desk at home dedicated to study). Find out when you operate at your peak, and endeavour to study at those times of the day. This might be between 8am and 10am or could be in the evening. Take short breaks during your study to remain focused – it's definitely time for a short break if you find that your brain is tired and that your concentration has started to drift from the information in front of you.

### **Order of study**

We suggest that you work through each of the chapters in turn. To get the maximum benefit from each chapter you should proceed in the following order:

1. Read the Syllabus Objectives. These are set out in the box at the start of each chapter.
2. Read the Chapter Summary at the end of each chapter. This will give you a useful overview of the material that you are about to study and help you to appreciate the context of the ideas that you meet.
3. Study the Course Notes in detail, annotating them and possibly making your own notes. Try the self-assessment questions as you come to them. As you study, pay particular attention to the listing of the Syllabus Objectives and to the Core Reading.
4. Read the Chapter Summary again carefully. If there are any ideas that you can't remember covering in the Course Notes, read the relevant section of the notes again to refresh your memory.
5. Attempt (at least some of) the Practice Questions that appear at the end of the chapter.

It's a fact that people are more likely to remember something if they review it several times. So, do look over the chapters you have studied so far from time to time. It is useful to re-read the Chapter Summaries or to try the Practice Questions again a few days after reading the chapter itself. It's a good idea to annotate the questions with details of when you attempted each one. This makes it easier to ensure that you try all of the questions as part of your revision without repeating any that you got right first time.

Once you've read the relevant part of the notes and tried a selection of questions from the Practice Questions (and attended a tutorial, if appropriate) you should attempt the corresponding assignment. If you submit your assignment for marking, spend some time looking through it carefully when it is returned. It can seem a bit depressing to analyse the errors you made, but you will increase your chances of passing the exam by learning from your mistakes. The markers will try their best to provide practical comments to help you to improve.

To be really prepared for the exam, you should not only know and understand the Core Reading but also be aware of what the examiners will expect. Your revision programme should include plenty of question practice so that you are aware of the typical style, content and marking structure of exam questions. You should attempt as many past exam questions as you can.

### **Active study**

Here are some techniques that may help you to study actively.

1. Don't believe everything you read. Good students tend to question everything that they read. They will ask 'why, how, what for, when?' when confronted with a new concept, and they will apply their own judgement. This contrasts with those who unquestioningly believe what they are told, learn it thoroughly, and reproduce it (unquestioningly?) in response to exam questions.
2. Another useful technique as you read the Course Notes is to think of possible questions that the examiners could ask. This will help you to understand the examiners' point of view and should mean that there are fewer nasty surprises in the exam room. Use the Syllabus to help you make up questions.
3. Annotate your notes with your own ideas and questions. This will make you study more actively and will help when you come to review and revise the material. Do not simply copy out the notes without thinking about the issues.
4. Attempt the questions in the notes as you work through the course. Write down your answer before you refer to the solution.
5. Attempt other questions and assignments on a similar basis, *ie* write down your answer before looking at the solution provided. Attempting the assignments under exam conditions has some particular benefits:
  - It forces you to think and act in a way that is similar to how you will behave in the exam.
  - When you have your assignments marked it is *much* more useful if the marker's comments can show you how to improve your performance under exam conditions than your performance when you have access to the notes and are under no time pressure.
  - The knowledge that you are going to do an assignment under exam conditions and then submit it (however good or bad) for marking can act as a powerful incentive to make you study each part as well as possible.
  - It is also quicker than trying to write perfect answers.

You can find further information on how to study in the UK Profession's Student Handbook, which you can download from their website at:

**[www.actuaries.org.uk/studying](http://www.actuaries.org.uk/studying)**

## Revision and exam skills

### *Revision skills*

You will have sat many exams before and will have mastered the exam and revision techniques that suit you. However it is important to note that due to the high volume of work involved in the Core Principles subjects it is not possible to leave all your revision to the last minute. Students who prepare well in advance have a better chance of passing their exams on the first sitting.

Unprepared students find that they are under time pressure in the exam. Therefore it is important to find ways of maximising your score in the shortest possible time. Part of your preparation should be to practise a large number of exam-style questions under timed exam conditions as soon as possible. This will:

- help you to develop the necessary understanding of the techniques required
- highlight the key topics, which crop up regularly in many different contexts and questions
- help you to practise the specific skills that you will need to pass the exam.

### *Exam question skill levels*

Exam questions are not designed to be of similar difficulty. The Institute and Faculty of Actuaries specifies different skill levels that questions may be set with reference to.

Questions may be set at any skill level:

- Knowledge – demonstration of a detailed knowledge and understanding of the topic
- Application – demonstration of an ability to apply the principles underlying the topic within a given context
- Higher Order – demonstration of an ability to perform deeper analysis and assessment of situations, including forming judgements, taking into account different points of view, comparing and contrasting situations, suggesting possible solutions and actions, and making recommendations.

## 2.1 Subject A213 – Syllabus and Core Reading

### Syllabus

The Syllabus for Subject A213 is given here. To the right of each objective are the chapter numbers in which the objective is covered in the ActEd course.

The numbering of syllabus objectives is based on the IFoA Subject CM1's syllabus, rather than the syllabus published on the ASSA's website.

### Aim

The aim of the Contingencies subject is to provide a grounding in the principles of modelling as applied to actuarial work – focusing particularly on deterministic models which can be used to model and value cashflows that are dependent on death, survival, or other uncertain risks.

### Competences

On the successful completion of this subject, the candidate will be able to describe, interpret and discuss mathematical techniques used to model and value cashflows which are contingent on mortality and morbidity risks.

### Syllabus topics

1.	Single decrement models	(20%)
2.	Multiple decrement and multiple life models	(20%)
3.	Pricing and reserving	(60%)

The weightings are indicative of the approximate balance of the assessment of this subject between the main syllabus topics, averaged over a number of examination sessions.

The weightings also have a correspondence with the amount of learning material underlying each syllabus topic. However, this will also reflect aspects such as:

- the relative complexity of each topic, and hence the amount of explanation and support required for it
- the need to provide thorough foundation understanding on which to build the other objectives
- the extent of prior knowledge which is expected
- the degree to which each topic area is more knowledge or application based.

### ***Skills level***

The use of a specific command verb within a syllabus objective does not indicate that this is the only form of question which can be asked on the topic covered by that objective. The Examiners may ask a question on any syllabus topic using any of the agreed command verbs, as are defined in the document “Command verbs used in the Associate and Fellowship written examinations”.

Questions may be set at any skill level: Knowledge (demonstration of a detailed knowledge and understanding of the topic), Application (demonstration of an ability to apply the principles underlying the topic within a given context) and Higher Order (demonstration of an ability to perform deeper analysis and assessment of situations, including forming judgements, taking into account different points of view, comparing and contrasting situations, suggesting possible solutions and actions, and making recommendations).

In the Contingencies subject, the approximate split of assessment across the three skill types is 20% Knowledge, 65% Application and 15% Higher Order skills.

### ***Detailed syllabus objectives***

4. Single decrement models (10%)

4.1 Define various assurance and annuity contracts. (Chapters 14, 15, 16, 18 and 25)

4.1.1 Define the following terms:

- whole life assurance
- term assurance
- pure endowment
- endowment assurance
- whole life level annuity
- temporary level annuity
- guaranteed level annuity
- premium
- benefit

including assurance and annuity contracts where the benefits are deferred.

4.1.2 Describe the operation of conventional with-profits contracts, in which profits are distributed by the use of regular reversionary bonuses, and by terminal bonuses. Describe the benefits payable under the above assurance-type contracts.

4.1.3 Describe the operation of conventional unit-linked contracts, in which death benefits are expressed as combination of absolute amount and relative to a unit fund.

- 4.1.4 Describe the operation of accumulating with-profits contracts, in which benefits take the form of an accumulating fund of premiums, where either:
- the fund is defined in monetary terms, has no explicit charges, and is increased by the addition of regular guaranteed and bonus interest payments plus a terminal bonus; or
  - the fund is defined in terms of the value of a unit fund, is subject to explicit charges, and is increased by regular bonus additions plus a terminal bonus (unitised with-profits).

In the case of unitised with-profits, the regular additions can take the form of (a) unit price increases (guaranteed and/or discretionary), or (b) allocations of additional units.

In either case, a guaranteed minimum monetary death benefit may be applied.

- 4.2 Develop formulae for the means and variances of the payments under various assurance and annuity contracts, assuming constant deterministic interest rate.  
(Chapters 14, 15, 16, 17, 18 and 20)

- 4.2.1 Describe the life table functions  $l_x$  and  $d_x$  and their select equivalents  $l_{[x]+r}$  and  $d_{[x]+r}$ .
- 4.2.2 Define the following probabilities:  ${}_n p_x$ ,  ${}_n q_x$ ,  ${}_n | m q_x$ ,  ${}_n | q_x$  and their select equivalents  ${}_n p_{[x]+r}$ ,  ${}_n q_{[x]+r}$ ,  ${}_n | m q_{[x]+r}$ ,  ${}_n | q_{[x]+r}$ .
- 4.2.3 Express the probabilities defined in 4.2.2 in terms of life table functions defined in 4.2.1.
- 4.2.4 Define the assurance and annuity factors and their select and continuous equivalents. Extend the annuity factors to allow for the possibility that payments are more frequent than annual but less frequent than continuous.
- 4.2.5 Understand and use the relations between annuities payable in advance and in arrear, and between temporary, deferred and whole life annuities.
- 4.2.6 Understand and use the relations between assurance and annuity factors using equation of value, and their select and continuous equivalents.
- 4.2.7 Obtain expressions in the form of sums/integrals for the mean and variance of the present value of benefit payments under each contract defined in 4.1.1, in terms of the (curtate) random future lifetime, assuming:

- contingent benefits (constant, increasing or decreasing) are payable at the middle or end of the year of contingent event or continuously.
- annuities are paid in advance, in arrear or continuously, and the amount is constant, increases or decreases by a constant monetary amount or by a fixed or time-dependent variable rate.
- premiums are payable in advance, in arrear or continuously; and for the full policy term or for limited period.

Where appropriate, simplify the above expressions into a form suitable for evaluation by table look-up or other means.

4.2.8 Define and evaluate the expected accumulations in terms of expected values for the contracts described in 4.1.1 and contract structures described in 4.2.7.

5. Multiple decrement and multiple life models (10%)

5.1 Define and use assurance and annuity functions involving two lives. (Chapters 21 and 22)

5.1.1 Extend the techniques of objectives 4.2 to deal with cashflows dependent upon the death or survival of either or both of two lives.

5.1.2 Extend the technique of 5.1.1 to deal with functions dependent upon a fixed term as well as age.

5.2 Describe and illustrate methods of valuing cashflows that are contingent upon multiple transition events. (Chapter 24)

5.2.1 Define health insurance, and describe simple health insurance premium and benefit structures.

5.2.2 Explain how a cashflow, contingent upon multiple transition events, may be valued using a multiple state Markov model, in terms of the forces and probabilities of transition.

5.2.3 Construct formulae for the expected present values of cashflows that are contingent upon multiple transition events, including simple health insurance premiums and benefits, and calculate these in simple cases. Regular premiums and sickness benefits are payable continuously and assurance benefits are payable immediately on transition.

5.3 Describe and use methods of projecting and valuing expected cashflows that are contingent upon multiple decrement events. (Chapter 24)

5.3.1 Describe the construction and use of multiple decrement tables.

5.3.2 Define a multiple decrement model as a special case of a multiple state Markov model.

5.3.3 Derive dependent probabilities for a multiple decrement model in terms of given forces of transition, assuming forces of transition are constant over single years of age.

5.3.4 Derive forces of transition from given dependent probabilities, assuming forces of transition are constant over single years of age.

6. Pricing and reserving (35%)

6.1 Define the gross random future loss under an insurance contract, and state the principle of equivalence. (Chapter 19)

6.2 Describe and calculate gross premiums and reserves of assurance and annuity contracts. (Chapters 19 and 20)

6.2.1 Define and calculate gross premiums for the insurance contract benefits as defined in objective 4.1 under various scenarios using the equivalence principle or otherwise:

- contracts may accept only single premium;
- regular premiums and annuity benefits may be payable annually, more frequently than annually, or continuously;
- death benefits (which increase or decrease by a constant compound rate or by a constant monetary amount) may be payable at the end of the year of death, or immediately on death;
- survival benefits (other than annuities) may be payable at defined intervals other than at maturity.

6.2.2 State why an insurance company will set up reserves.

6.2.3 Define and calculate gross prospective and retrospective reserves.

6.2.4 State the conditions under which, in general, the prospective reserve is equal to the retrospective reserve allowing for expenses.

6.2.5 Prove that, under the appropriate conditions, the prospective reserve is equal to the retrospective reserve, with or without allowance for expenses, for all fixed benefit and increasing / decreasing benefit contracts.

6.2.6 Obtain recursive relationships between successive periodic gross premium reserves, and use this relationship to calculate the profit earned from a contract during the period.

6.2.7 Outline the concepts of net premiums and net premium valuation and how they relate to gross premiums and gross premium valuation respectively.

6.3 Define and calculate, for a single policy or a portfolio of policies (as appropriate):

- death strain at risk;
- expected death strain;
- actual death strain; and
- mortality profit

for policies with death benefits payable immediately on death or at the end of the year of death; for policies paying annuity benefits at the start of the year or on survival to the end of the year; and for policies where single or non-single premiums are payable. (Chapter 23)

6.4 Project expected future cashflows for whole life, endowment and term assurances, annuities, unit-linked contracts, and conventional/unitised with-profits contracts, incorporating multiple decrement models as appropriate. (Chapters 26 and 27)

6.4.1 Profit test life insurance contracts of the types listed above and determine the profit vector, the profit signature, the net present value, and the profit margin.

6.4.2 Show how a profit test may be used to price a product, and use a profit test to calculate a premium for life insurance contracts of the types listed above.

6.4.3 Show how gross premium reserves can be computed using the above cashflow projection model and included as part of profit testing.

6.5 Show how, for unit-linked contracts, non-unit reserves can be established to eliminate ('zeroise') future negative cashflows, using a profit test model. (Chapter 27)

## Assessment

Combination of a one hour 45 minute computer based modelling assignment and a two hour written examination.

## **Core Reading**

The Subject A213/CM1 Course Notes include the Core Reading in full, integrated throughout the course.

### ***Accreditation***

The Institute and Faculty of Actuaries would like to thank the numerous people who have helped in the development of the material contained in the Core Reading.

### ***Further reading***

The exam will be based on the relevant Syllabus and Core Reading and the ActEd course material will be the main source of tuition for students.