Subject SP9

CMP Upgrade 2022/23

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2022 CMP to make it suitable for study for the 2023 exams. It includes replacement pages and additional pages where appropriate.

Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our 2023 *Student Brochure* for more details.

We only accept the current version of assignments for marking, *ie* those published for the sessions leading to the 2023 exams. If you wish to submit your script for marking but only have an old version, then you can order the current assignments free of charge if you have purchased the same assignments in the same subject in a previous year, and have purchased marking for the 2023 session.

This CMP Upgrade contains:

- all significant changes to the Syllabus and Core Reading
- additional changes to the ActEd Course Notes and Assignments that will make them suitable for study for the 2023 exams.

0 Changes to the Syllabus

There are no changes to the syllabus objectives.

1 Changes to the Core Reading and ActEd text

This section contains all the ***non-trivial*** changes to the Core Reading and ActEd text.

Module 1

Module Summary

The definition of risk has been deleted from the Module Summary. A broader discussion of risk *vs* uncertainty is now in Module 3 (see below).

Module 2

Section 1

The solution to the question on the benefits of ERM has been modified to reflect points made in recent exam solutions. A replacement page is provided.

Module 3

Syllabus objective 3.1 has been moved to Module 3.

Section 1 and Module Summary

A discussion of risk *vs* uncertainty has been added. Lam's risk concepts have also been moved to this section from Module 13. Replacement pages are provided.

Module 13

Section 1

A new question and solution on the contents of a business plan have been added. A replacement page is provided.

Section 3, Module Summary and End of Module Q3.1

Lam's risk concepts have been removed (now in Module 3). End of Module Q3.1 has been deleted.

Module 14

Section 4 and Module Summary

The material on the choice of time horizon has been rewritten. Replacement pages are provided.

Module 15

Section 3

A new section of Core Reading entitled, ‘Further thoughts on scenario analysis and stress testing’ has been added, along with an ActEd question. Replacement pages are provided.

End of Module Questions

A new question, Q15.2, has been added covering sensitivity testing, scenario analysis and stress testing. Replacement pages are provided.

Module 16

Section 2

Two-thirds down Page 21, the statement should read:

$\bar{\mathbf{X}}$ is an unbiased estimator for α.

The material on Cholesky Decomposition and Principal Components Analysis (PCA) has been modified to improve the notation used and to include a recap as to how to obtain the eigenvalues and eigenvectors of a matrix. Replacement pages are provided.

Module Summary

On the second page of the Module Summary, in the section on the *t*-distribution, the explanation of the power law has been modified (3rd bullet). References to *s* being the sample variance have been changed to the sample standard deviation (5th and 6th bullet points). A replacement page is provided.

Module 17

Section 2 and Module Summary

The condition for an $AR(p)$ time series and an to be covariance stationary has been modified to:

For the $AR(p)$ process described above to be covariance stationary, all roots (z) of the following characteristic polynomial expression must lie outside the unit circle:

$$f(z) = 1 - \alpha_1 z - \alpha_2 z^2 - \ldots - \alpha_p z^p = 0$$

Section 3 and Module Summary

The condition for an $ARCH(p)$ time series and an to be covariance stationary has been modified to:

For an $ARCH(p)$ model to be weakly stationary, the roots (z) of the characteristic polynomial expression, $f(z) = 1 - \alpha_1 z - \alpha_2 z^2 - \ldots - \alpha_p z^p = 0$, must lie outside the unit circle.

Module 21

End of Module Questions

Q21.4(ii) has been removed (now in Module 15).

Module 23

Section 1

In the material on 'Obtaining information', the reference to 'moral hazard' has been replaced with 'anti-selection':

Sections 3 and 4

Some updates have been made to the material on the Merton, KMV and CreditMetrics models, to clarify the advantages and disadvantages. In particular, the question on the CreditMetrics model now focuses on technical advantages and disadvantages, whilst an assignment question focuses on practical advantages and disadvantages. Replacement pages are provided.

Module 24

Section 2

The material on scenario analysis has been modified to provide better consistency / linkage with Module 15. A replacement page is provided.

Module 29

Section 1

The solution to the question on model and data risk has been expanded. A replacement page is provided.

Module 30

Sections 0 and 1, and Module Summary

The material introducing the module and on capital terminology have been rewritten. Replacement pages are provided.

New Section 3

A new section on 'Capital as a management tool' has been added just before the original Section 3 on Risk optimisation. A replacement page is provided.

Section 4

The ActEd text accompanying the Core Reading examples has been modified in several places to help provide clarity. Replacement pages are provided.

Module 32

Section 1

The Core Reading on the covid-19 case study has been modified – introductory paragraphs only. A replacement page is provided.

Module 34

Section 1

Various URLs have been updated, where these had broken. A link to The Bank of England's Biennial Exploratory Scenario has been added. Replacement pages are provided.

2 Changes to the X Assignments

Changes to the X Assignments are listed below.

Assignment X1

No changes

Assignment X2

No changes

Assignment X3

Question X3.1 has been rewritten to make it more 'applications' based. A new question, X3.2, on Cholesky decomposition and Principal Components Analysis (PCA) has been added. Replacement pages are provided for both the assignment questions and solutions.

Assignment X4

Question X4.3 has been modified to focus on the *relative practical* advantages of structural and credit migration models. The question now reads:

Discuss the relative *practical* advantages and disadvantages of each of the following types of model used to estimate credit risk:

- structural (or firm-value) models
- credit migration (or reduced-form) models.

A replacement page is provided for the solution to this question.

Assignment X5

No changes

Assignment X6

No changes

3 Other tuition services

In addition to the CMP you might find the following services helpful with your study.

3.1 Study material

We also offer the following study material in Subject SP9:

- Flashcards
- ASET (ActEd Solutions with Exam Technique) and Mini-ASET
- Mock Exam and AMP (Additional Mock Pack).

For further details on ActEd's study materials, please refer to the 2023 *Student Brochure*, which is available from the ActEd website at **ActEd.co.uk**.

3.2 Tutorials

We offer the following (face-to-face and/or online) tutorials in Subject SP9:

- a set of Regular Tutorials (lasting a total of three days)
- a Block Tutorial (lasting three full days).

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **ActEd.co.uk**.

3.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to the 2023 *Student Brochure*, which is available from the ActEd website at **ActEd.co.uk**.

3.4 Feedback on the study material

ActEd is always pleased to receive feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course, please send them by email to **SP9@bpp.com**.

This enables them to:

- better understand the organisation's aggregate risk exposure, *ie* allowing for interdependencies
- better comprehend the links between business growth, corporate risk and return
- better understand the impact of changing external factors, such as interest rates
- assess more accurately the risk / return trade-offs of a particular decision
- align strategy more closely with risk appetite.

Senior management are better informed because a central risk function can take responsibility for risk reporting across the whole organisation, ensuring that *all* risks are reported in a *consistent* and appropriate format to stakeholders – increasing *risk transparency*.

A central risk function can also improve an organisation's *operational effectiveness* by:

- co-ordinating risk management activities across all parts of the organisation
- encouraging and facilitating the sharing of risk information
- identifying and assessing interdependencies between risks managed by various teams
- improving efficiency (*eg* with respect to management time and business resources).

As a consequence, ERM may enhance an organisation's *business performance* by:

- using and allocating capital more efficiently
- minimising losses and unpleasant surprises
- reducing earnings volatility
- incorporating risk into business processes, *eg* pricing, business acceptance
- optimising risk transfer and other risk management decisions, *eg* by allowing for natural hedges between business units
- reacting more quickly, *eg* seizing opportunities, detecting emerging risks earlier
- deriving value from the time, effort and money spent on risk management, rather than it being viewed as a box-ticking exercise.

The increase in widely-available tools to assess and manage risk has increased stakeholders' expectations that companies will manage risk effectively. ERM may help reduce the risk of regulatory intervention as well as have a positive impact on an organisation's credit rating.

The integration of ERM into business decision-making processes in order to create value is considered further in Module 8.

Question

Outline what pressures might cause an organisation to *initiate* an ERM programme.

Solution

Companies find that pressure to implement an ERM programme often emerges in light of:

- previous management failures
- a 'near miss' within their own organisation
- a high-profile disaster in another similar organisation
- criticism or demands from a regulatory body or auditor
- concerns from (other) stakeholders.

The case studies in Chapter 21 of Lam are intended to illustrate how that ERM adds value to an organisation.

Question

Outline the key points from the case studies that support this assertion of ERM adding value.

Solution

The key points from the case studies are:

- Many investors would avoid a company, or a country, with poor governance standards.
- Investors are willing to pay a premium for well-governed companies (anywhere from 12% to 30%).
- Companies with strong governance structures (to be discussed in Module 4) outperform those with weaker governance (as measured by return on equity, profit margins and dividends).
- The effect is amplified for larger companies.
- Specifically, insurance companies with ERM programmes have lower volatility of returns, improved shareholder value, financial stability and a 16% equity premium.
- Stock price performance (returns and volatility) for those companies with an 'excellent' Standard and Poor's ERM rating (to be discussed in Module 7) was better than those of 'weak' companies during the stock market crashes of 2008.

1 Introduction to risk

1.1 Risk *vs* uncertainty

There is no single definition of *risk* that is applicable to all circumstances.

The words *risk* and *uncertainty* are often used interchangeably in everyday language. However, as discussed in Subject CP1, they are different concepts.

- Typically, we might describe ourselves as being in a state of uncertainty if we lack knowledge / understanding of the processes which leads to the various possible outcomes (*eg* we might not know which dependent variables to use in our model and/or which formulae to use). Alternatively, we might describe ourselves as being in a state of uncertainty because we are not able to identify all the possible material outcomes which could arise (*eg* as part of a scenario analysis exercise). In such states of uncertainty, we will be unable to model the potential outcomes.
- Alternatively, if we can produce a model, then we might say we are exposed to risk as opposed to uncertainty. If, further, we have credible data and are in a position to make credible assumptions (*eg* identify an appropriate probability distribution for losses), then we can quantify / estimate that risk (*ie* it is a quantifiable / estimable risk). However, if we do not have credible data and/or are not able to make credible assumptions on the back of that data, then we are faced with an unquantifiable / inestimable risk. In the latter case, we might still be able to assess the risk, *eg* as being low, medium, or high ranking.

Building on the above, if we are faced with a risk (rather than an uncertainty) then, we understand the processes leading to the potential outcomes, and so are in a stronger position to manage (*eg* mitigate) that risk, as well as to decide whether or not to take the risk. Conversely, if we are faced with an uncertainty, then we are not in such an empowered position.

So, some aspects of climate change might be described as risks – for example, we might feel relatively confident in modelling relatively short-term changes to air temperatures and sea levels. However, more broadly, climate change puts us in a state of uncertainty, in that we cannot claim to have full knowledge / understanding of the processes involved, nor of the full range of possible outcomes.

Question

Explain how a car journey from destination A to destination B exposes the driver to risk.

Solution

A car journey from A to B exposes the driver to risk as we can define the outcomes and estimate the probabilities, for example:

- arrive at B, estimated probability 0.99
- don’t arrive at B because of a breakdown, estimated probability 0.005
- don’t arrive at B because of an accident, estimated probability 0.005.

We can model the risk in terms of a probability of each outcome, and/or a severity of the loss associated with each outcome (*eg* cost of fuel, repair, damage).

We can manage the risks associated with the two adverse outcomes, for example by buying insurance, by ensuring the car is regularly serviced and by driving safely.

We can choose whether to take the risk by choosing whether or not to undertake the car journey.

However, as well as risk, there is also uncertainty attached to the car journey. For example, there may be outcomes that we have not even thought about, or outcomes where we cannot estimate the probability because of a complete lack of knowledge of its likelihood.

1.2 Risk concepts

Lam discusses seven *risk concepts*:

1. exposure
2. volatility
3. probability
4. severity
5. time horizon
6. correlation
7. capital.

All other things being equal, the larger any of the first six items are, the greater the risk.

Question

Outline each of the seven risk concepts discussed in Lam.

Solution

1. Exposure – the maximum loss that can be suffered if an event occurs.

   Exposure may be qualitative as well as quantitative, *eg* damage to brand name.

2. Volatility – broadly, a measure of the variability within the range of possible outcomes.

   When describing market risk, volatility is defined as the standard deviation of returns.

3. Probability – the likelihood that an event occurs.

4. Severity – the loss that is likely to be incurred if an event occurs.

   Severity is generally lower than exposure (which is the maximum loss).

Multiplied together, probability and severity give a useful assessment of the expected cost of the risk.

5. Time horizon – the length of time for which an organisation is exposed to a risk, or the time required to recover from (or reverse the effects of) an event.

6. Correlation – the degree to which differing risks behave similarly in response to common events.

   A high correlation indicates a concentration of risk, a low correlation indicates diversification.

7. Capital – the money set aside to cover unexpected losses (risk capital) and to support ongoing business strategy (working capital) such as writing new business, development of a new product.

Sweeting also refers to many of these concepts, as well as the concept that risk can refer to the problems (downside risk) and opportunities (upside risk) that arise from outcomes not being as expected.

2 Risk taxonomy

2.1 Definition

A risk taxonomy is a full list, description and categorisation of all risks that an organisation might face. There is no single universally accepted risk taxonomy.

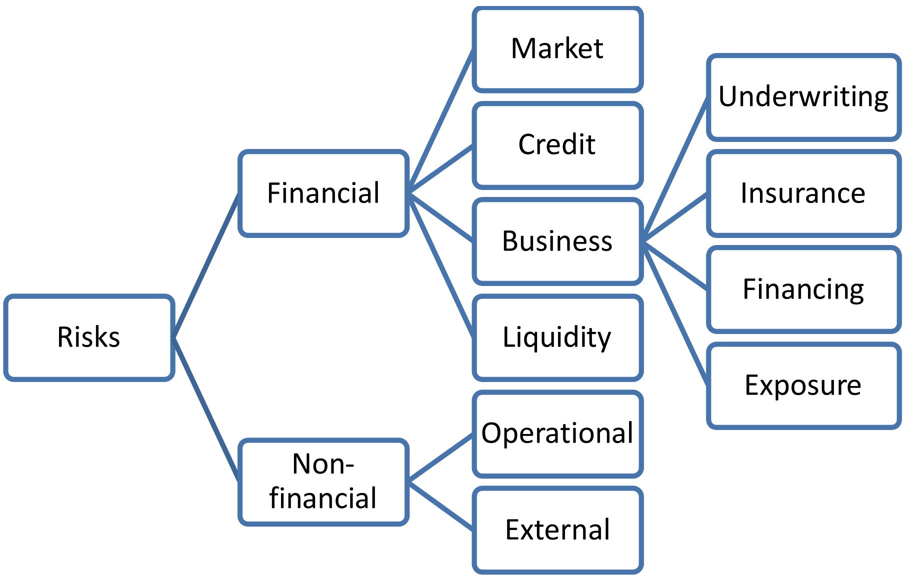
2.2 High-level risk categorisations

In a financial or insurance context, a common, high-level risk categorisation (as seen in the Basel or Solvency II frameworks introduced in Module 5) is made up of five risk types:

1. *market risk* – risks arising from changes in market values
2. *liquidity risk* – risks associated with funding or cashflow requirements
3. *credit (or default or counterparty) risk* – the risk of failure of a third party to meet contractual obligations in a timely way
4. *operational risk* – the risk of loss resulting from deficient internal processes, people or systems or loss due to external events
5. *underwriting / insurance risk* – the risk of accepting risks which turn out to be inappropriate or pricing accepted risks inappropriately.

Market, liquidity and credit risks are commonly described grouped under the umbrella category of *financial* risks.

Recall that Subject CP1 introduced a possible categorisation of risk for a financial services provider:



Module 3 Summary – Risk taxonomy

Risk *vs* uncertainty

- Risk refers to the situation where all possible outcomes and their probabilities are known (or, at least, can be estimated). Risk can typically be modelled and managed.
- Uncertainty refers to the situation where we lack knowledge / understanding of the processes which leads to the various possible outcomes. Uncertainty cannot be modelled and is very difficult to manage.

Risk concepts

1. exposure
2. volatility
3. probability
4. severity
5. time horizon
6. correlation
7. capital.

Risk taxonomy

A risk taxonomy is a full list, description and categorisation of all risks that an organisation might face. There is no single universally accepted taxonomy for risks, however, most companies will consider market, liquidity, credit and operational risks, as well as risks specific to their business (in particular, insurance risks for insurance companies).

Market risk

Market risk is the risk arising from changes in market values or other features correlated with market values such as interest and inflation rates. It includes the consequences of asset value changes on liability values and asset-liability mismatching.

The term market risk might alternatively be used to the risk arising from changes in the market into which products or services are sold (*eg* lower sales).

Liquidity risk

Funding liquidity risk is the inability to obtain funds to meet expected and unexpected current obligations as they fall due (or at least without excessive cost) or, more broadly, an inability to manage of short-term cashflow requirements.

Market liquidity risk is insufficient capacity in the market to handle asset transactions at the time when the deal is required (without a material impact on price).

Credit risk

Credit risk is the risk that a counterparty to an agreement will be unable or unwilling to make the payments required (in full or in part) under that agreement.

Operational risk

Operational risk is risk resulting from inadequate or failed internal processes, people and systems, or from failure to recover from external events.

Insurance risk

Insurance risk arises from fluctuations in the timing, frequency and severity of insured events, relative to the expectations at the time of underwriting or pricing. It includes mortality, morbidity, property and casualty risks.

Other risks

Other risks mentioned in the SP9 syllabus:

- economic risk
- interest rate risk
- foreign exchange risk
- basis risk
- counterparty risk
- environmental risk
- legal risk
- regulatory risk (or compliance risk)
- political risk
- agency risk
- reputational risk
- project risk
- strategic risk
- demographic risk
- moral hazard
- cyber risk
- climate change risk.

Systematic and non-systematic risks

Systematic risk is risk that cannot be (fully) diversified.

Non-systematic risks (or *specific* risks) are risks that are uncorrelated with, or possibly independent from, other sources of risk. A risk that is typically non-systematic may, under stressed conditions become systematic.

Concentration of risk

A concentration of risk occurs as a consequence of a lack of diversity in risk exposures. It can be described as 'putting all your eggs in one basket', that is, relying upon the success of one course of action which, if it fails, leaves no alternative

Systemic risk and contagion risk

Systemic risk is the risk of failure of a system due to problems spreading between inter-dependent entities within the system.

Financial contagion risk is a type of systemic risk where financial losses in one company, industry sector or country lead to losses in another.

The practice questions start on the next page so that you can keep the module summaries together for revision purposes.

1 The risk identification and assessment process

In order to identify and assess the risks it faces a company should have a well-defined process.

Question

Describe a six-step process designed to produce and maintain a comprehensive identification and initial assessment of the risks faced by a business.

Solution

1. Business analysis:
   - Ensure that the company has clear business objectives. (If it is unclear what the company's goals are, it is difficult to establish what risks can impact on their achievement.)
   - Analyse its operations and its wider environment. This business analysis will involve looking at:
     – the business plan (*see next question*)
     – the company's structure and its system of internal controls
     – current and projected accounts, and accounting ratios
     – market information, such as competitors' actions and market share
     – the resources available to the company
     – legislative and regulatory constraints
     – the general economic environment.

2. Identify the risks (upside and downside) the company faces in a structured way. The starting point for this will be a review of the findings of the business analysis to identify any areas of risk, uncertainty or opportunity.

   Possible methods for risk identification are considered in Section 2.

3. Obtain agreement on the risks faced, the relationships between them, and identify individuals who will be responsible for each risk and its management.

4. Evaluate the risks in terms of their likelihood of occurrence (*eg* probability of occurrence within one year) and severity of impact (financial, strategic and reputational consequences). This might be done both gross and net of existing controls. This enables risks to be prioritised for further implementation of controls.

5. Produce a risk register to record the results of this process in one place.

   We will look further at the elements of the risk register in Section 2.

6. Review the risk register regularly, and especially in times of change, to ensure that it remains up-to-date and reflects the current risks faced by the company.

Question

Outline the key components of a business plan.

Solution

- statement of business objectives
- description of business, products and services
- description of economic environment, including industry sector, regulation, customers, suppliers and competitors
- identification of perceived key risks, including upside risks / opportunities
- description of strategy, including:
  - choice of opportunities to be pursued
  - marketing plan
  - operating plan, including expected resourcing / capital requirements
  - planned risk responses
- description of organisational structure
- forecast of expected financial outcomes
- statements of key assumptions, and sensitivity of expected outcomes to these

Benefits of risk identification and assessment

Investing time and resources in risk identification and assessment is an important first step in the risk management process, as it:

- enhances awareness and transparency of risks in the organisation
- helps transfer knowledge and improve understanding across the organisation
- acts as a firm base for subsequent risk analysis, quantification and prioritisation
- enhances the quality of reporting to the Board and senior management
- hence, helps improves business decision making.

Requirements

In order to gain these benefits, it is necessary for an organisation to have senior sponsorship of the risk management programme.

Question

Outline other necessary conditions for an organisation to gain the above benefits of risk identification and assessment.

4 Risk management time horizon

Time horizon was identified earlier (in Module 3) as being one of seven key risk concepts. It was described as being:

- the length of time for which an organisation is exposed to a risk, or
- the time required to recover from (or reverse the effects of) an event.

The longer the duration of exposure, the higher the level of risk – both in terms of the possible outcome (*eg* insolvency) and what might happen in the intervening period (*eg* liquidity problems).

Question

Describe the key factors affecting the choice of a suitable time horizon.

Solution

The choice of time horizon will be influenced by expectations as to the time period over which an organisation is committed to holding its risk portfolio (and hence the time exposed to the risk). This in turn is affected by:

- contractual / legal constraints, *eg* a general insurance company is typically bound to a claims portfolio for one year
- liquidity considerations, *ie* the time taken to liquidate an investment portfolio, which, in adverse conditions, may be longer than is usual
- the time to reinstate risk mitigation, *eg* to re-establish a derivatives hedge
- the time to recover from a loss event, *eg* for operational risks such as fire.

5 Risk discount rate

5.1 Project appraisal

The following are extracts from the publication *Risk analysis and management for projects (RAMP): a strategic framework for managing project risk and its financial implications*, jointly published by the Institution of Civil Engineers and the Institute and Faculty of Actuaries:

'The size of the discount rate will affect the appraised viability of those projects to which it is applied: broadly, the higher the discount rate the lower will be the present value of earnings (or benefits) arising in the future and the greater the negative impacts on project viability. The discount rate is determined pragmatically by the sponsor. Ideally it should take account (among other things) of the sponsor's cost of capital, the rate of inflation, interest rates and rates of return on investments throughout the economy.'

However:

- Suitable reference investments may not exist (*eg* there may not be a risk-free asset that matches certain liabilities).
- It may be difficult to determine the risk-free rate of interest (*eg* due to government securities benefiting from a liquidity premium).
- Setting a discount rate that allows for the uncertainty of future asset values is problematic (*eg* a projection/simulation approach may be required).
- An allowance for credit (*eg* default) risk should be made when setting the discount rate and default rates are linked to other risks which vary over time (*eg* inflation risk).

'Ultimately the choice of the discount rate will depend partly on issues such as the company's cost of funds and any hurdle rates that the company sets for its investments. Some companies may wish to use a higher / lower discount rate for projects which they regard as having a higher / lower inherent risk (*ie* a risk which is incapable of mitigation) than for their other projects.

If this inherent risk varies significantly over different phases of the project, it may sometimes be appropriate to use different discount rates for each phase.

A high discount rate should not be seen as a substitute for a detailed risk analysis as this could lead to the rejection of profitable low risk projects in favour of more profitable projects that carry unacceptable levels of risk.'

Module 14 Summary – Introduction to risk measurement

Coherence

The axioms of coherence are properties that a good risk measure should have. These are:

- monotonicity
- subadditivity
- positive homogeneity
- translation invariance.

A consequence of subadditivity and positive homogeneity is convexity, which expresses the benefits of diversification.

Types of risk measure

The two major types of risk measure are:

1. deterministic
2. probabilistic.

A deterministic risk measure gives a broad indication of the level of the risk taken. Examples are:

- notional approach
- factor sensitivity approach
- scenario sensitivity approach.

A probabilistic risk measure involves applying a statistical distribution to a risk (risks) and measuring a feature of that distribution. Examples are:

- deviation measures (*eg* standard deviation, tracking error, information ratio)
- Value at Risk (VaR)
- ruin probability
- Tail Value at Risk (TVaR)
- expected shortfall (ES).

VaR, TVaR and expected shortfall can be calculated using empirical, parametric and stochastic approaches.

Choosing a time horizon

The choice of time horizon will be influenced by expectations as to the time period over which an organisation is committed to holding its portfolio (and hence the time exposed to the risk). This in turn is affected by:

- contractual / legal constraints
- liquidity considerations
- the time to reinstate risk mitigation
- the time to recover from a loss event.

Choosing a risk discount rate

The factors to consider when setting a discount rate for a project include:

- the organisation's cost of capital
- the level of inherent risk exposure
- inflation rates
- interest rates
- investment returns in the economy.

In theory, the Capital Asset Pricing Model (CAPM) can be used to assess a discount rate to be used for valuing investments (and projects) using the security (project) market line.

2. 'bottom up' stress-variable tests, where, instead of looking at a particular scenario and varying all risk factors in a mutually-consistent fashion, the effect of a significant adverse change in a crucial factor (or a narrow range of crucial factors) is considered.

The process involves: deciding which stresses to consider, how extreme they are to be, and then running the risk models to quantify the impact (both pre- and post-potential mitigation strategies).

Question

Outline the advantages and limitations of stress testing.

Solution

The advantages of stress testing include:

- the ability of supervisors to compare the impact of the same stresses on differing organisations
- the explicit examination of extreme events which might not otherwise be considered, *eg* if a stochastic approach was adopted
- use in assessing the suitability of any response strategies, by assessing the expected (gross) impact of the stress in the absence of any response, and then the expected (net) impact in the presence of the proposed response.

The limitations include:

- It is subjective as to which assumptions to stress and the degree of stress(es) to consider.

  There may be some standard stress tests issued by supervisors to guide the process, but it is imperative that the company modifies or augments these to suit its own individual circumstances.
- It assigns no probability to the events considered.
- It looks only at extreme situations, and so needs to be coupled with other techniques, *eg* simulation, in order to understand the full range of outcomes.

3.2 Further thoughts on scenario analysis and stress testing

Scenario analysis and stress testing are powerful techniques which are used widely in a number of applications of Enterprise Risk Management. A very important part of a risk department's role is to ask the question, 'What if …?', and these techniques are useful.

In reviewing the material required for this section, candidates should bear the following in mind:

1. **Terms such as 'scenario analysis', 'stress testing', 'scenario testing' and 'stress scenario testing' are used almost interchangeably, and different authors and practitioners use the same term to mean different things. As always, where there is potential for confusion, it is advisable to state explicitly what meaning has been attached to specific terms.**

We have differentiated between the terms sensitivity analysis, scenario analysis and stress testing above, to reflect the distinctions drawn out in the textbooks.

2. Some authors try and make a distinction between hypothetical and historical scenarios. In terms of analysis, the distinction is moot – the risk practitioner's role is to identify a scenario and then assess its impact. However, in terms of communication there can be a useful distinction to be drawn, to persuade those responsible for managing risks that the extreme consequences modelled are in fact plausible – because in a real historical scenario they would actually have happened.

The point being made here is that it can help to convince the sceptics if the risk practitioner can illustrate that certain consequences have happened historically.

3. Although covered in this module, which covers risk modelling, the techniques are not solely quantitative. Scenario analysis is a core technique in understanding and managing operational Risk.

A broader application of scenario analysis, to operational risk modelling, is discussed in Module 24.

4. A full analysis requires consideration not just of the financial and quantitative consequences of the stress or scenario, but also of the practical implications:

- **Will there be operational strain?**
- **Will there be regulatory interest?**
- **Will there be consequences for dividend affordability?**
- **Will there be practical consequences for customers?**

… and so on

Question

An insurance company is carrying out a scenario analysis where the scenario under consideration is a recession. Give examples of how the above practical implications may arise.

Solution

Operational strain – in a recession, the insurance company may be forced to make some staff redundant, putting more pressure on the remaining employees. In turn, this may result in more errors and customer complaints. Similarly, there may be less investment in technology and systems during a recession, leading to increased likelihood of system failures.

Regulatory interest – as risk limits are reached (*eg* falls in stock markets, credit losses), the regulator may be interested in whether these risks are being monitored and action taken when risk limits are reached.

The regulator will also be interested in the stability of the insurance sector as a whole and concerned about systemic risk.

Dividend affordability – a fall in profits, or a desire to retain rather than distribute profits to support an organisation during a recession, may result in a reduction in dividends.

Practical consequences for customers – customers may no longer be able to afford to pay premiums in a recession. There may be a change in customer behaviour, *eg* more fraudulent claims.

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5. **Nevertheless, as always in actuarial work, the analysis should be appropriate and proportionate to the requirement. A full analysis including operational consequences is useful for a Board risk committee review, but something simpler and less resource-intensive could be sufficient to check that exposures are within adequate bounds.**

6. **The accuracy of the analysis depends on the accuracy of the inputs and the data used. Where there are known data inadequacies (*eg* the specific instruments held in third-party managed investment funds without look-through) these should be appropriately allowed for in the analysis and its reporting.**

'Without look through' means the investor will not know the details of the specific assets held by the fund.

There are many potential uses of stress and scenario tests, and the specific use should be borne in mind as the analysis is defined. Careful design may mean that the same piece of analysis can be used in multiple ways. A stress or scenario test may be used as part of:

- **an exercise that seeks to identify the key risks to which the firm is exposed**
- **an exercise seeking to understand whether the firm is being managed within the Board's risk appetite (either formally articulated or generally understood)**
- **a formal limit framework – a limit may be set so that exposures are managed in such a way that in a specified adverse scenario they are still within tolerance**
- **regulatory oversight – regulators sometimes ask all or many firms in a particular sector to complete the same scenario analysis, either to ensure that individual firms are managing key risks adequately or to assess the systemic exposure of the sector to a specific risk.**

3.3 Links to other processes

Business Continuity Management

The process of scenario analysis has clear links with *Business Continuity Management* (BCM), a program which exists in most businesses to ensure that they can continue to operate in the face of disaster or extreme events, usually in the context of operational risk. The idea from BCM of simulating emergencies to test what participants' reactions would be, can be a useful way of establishing the responses to an extreme event, which scenario analysis will then use to determine their likely long-run impact.

Back-testing

Back-testing is a way of validating the models currently in use within an organisation. It involves running a model using historic data (so we effectively use a scenario that has already happened), and comparing the model output with what actually happened in reality. Any discrepancies can be investigated and their causes remedied. Back-testing of models is required under Basel II, and the results of this can impact on a bank's capital requirements.

4 Stochastic modelling

A stochastic model is used when the inputs to a model are uncertain. Its key benefit is that it provides a probability distribution for the model outputs.

This is achieved by running the model repeatedly, each run being known as a *simulation*, and accumulating the results of these simulations to give a distribution of potential outcomes. From this outcome distribution, we can estimate the mean outcome, its variance, and probabilities associated with the outcome being more or less than a certain value.

For example, if 200 out of a total of 10,000 simulations lead to an outcome that exceeds £10 million, then we would estimate that the probability that the outcome exceeds £10 million is 0.02.

4.1 Historical simulation (Bootstrapping)

Under a historical simulation approach, each simulation is generated through direct reference to historical data, *eg* by random sampling.

Question

Outline the advantages and disadvantages of historical simulation.

Solution

The advantages of the historical simulation approach include:

- It is applicable to many situations, as long as suitable past data is available.
- It does not require large amounts of past data, if the sampling is done 'with replacement'.
- It does not require the specification of probability distributions for the inputs.
- It reflects the characteristics of the past data (including non-linearity, non-normality, interdependencies *etc*) without the need for parameterisation.

The disadvantages include:

- It cannot be performed in the absence of any relevant past data.
- It assumes that past data is indicative of the future.
- It does not take into account inter-temporal links between past data items (*eg* auto-correlations).
- It may underestimate uncertainty (as it is based only on what actually happened in the past, rather than on what potentially could have happened) and so, in practice, should generally be used with other methods (*eg* stress tests) so as to consider a greater range of outcomes.

Module 15 Practice Questions

15.1 You are investigating the relationship between equity returns and bond default rates. You have obtained the following data:

Exam style

Year	*Bond default rate*	*Equity return*
2016	3.40%	–7.60%
2015	0.40%	25.20%
2014	3.80%	4.20%
2013	1.20%	6.30%
2012	1.80%	–1.50%
2011	0.30%	12.60%
2010	0.45%	13.20%
2009	1.10%	1.20%
2008	4.50%	3.20%
2007	3.50%	7.30%

Calculate the correlation between these two datasets using:

(i) Pearson's rho

(ii) Spearman's rho

(iii) Kendall's tau. [9]

15.2 You are employed as an ERM consultant to a medium-sized manufacturing company. As part of your risk review, you are looking at the financial risks associated with the company's final salary pension scheme.

Exam style

The liabilities of the pension scheme are currently valued using a model with the following best estimate (or realistic) inputs:

Interest rate: 6% *pa*

Inflation: 3% *pa*

Salary growth: 4% *pa*

Mortality: CMI2018 – 2 years, 1.25% *pa* mortality improvements

Proportion of members married at retirement: 90% (used to make allowance for dependants' benefits).

The inflation assumption is used to allow for growth in deferred pensions in the period from withdrawal from the pension scheme to retirement.

In retirement, annual pension increases are awarded in line with the rate of inflation, subject to a minimum annual increase of 4%.

Describe how you would carry out each of the following on the model for the pension scheme liabilities:

(a) sensitivity analysis

(b) scenario analysis

(c) stress testing.

You should include examples of the scenarios or tests you would run. [11]

2010	0.45%	13.20%	D	D	D	D	D	C				
2009	1.10%	1.20%	D	D	C	C	D	D	D			
2008	4.50%	3.20%	C	D	D	D	C	D	D	C	
2007	3.50%	7.30%	C	D	D	C	C	D	D	C	D

Where: D indicates a discordant pair and C indicates a concordant pair.

So: $p_c - p_d = 13 - 32 = -19$ [2 for table and calculation of $p_c - p_d$]

and $t_{X,Y} = \dfrac{2}{10(10-1)}(-19) = -0.422$ [1]

[Total 3]

15.2 Methods of varying inputs and examples

(a) *Sensitivity analysis*

Sensitivity analysis involves varying each assumption one at a time to understand how each affects the total liability. [1]

No probability of occurrence is assigned to each of the possibilities. [½]

It might not be carried out for every input – just the key inputs. [½]

Examples:

- Assess the cost of a $\pm 1\%$ change in interest rates. [½]
- Assess the cost of a $\pm 1\%$ change in salary growth. [½]
- Assess the cost of the inflation assumption changing to 4%, 5% and 6%. It is necessary to consider the inflation assumption changing by more than 1%, as a 1% change from its current value would have no effect on the pension increase assumption. [1]
- Assess the cost of a change in the age rating for mortality by ± 1 years. [½]
- Assess the cost of a change in the (base) mortality table used. [½]
- Assess the cost of a change in mortality improvements by $\pm 0.25\%$ [½]
- It is likely that the assumption for the proportion of members married at retirement would not be sensitivity tested, as it has a minor effect when compared to the other assumptions. [½]

[Maximum 2 for appropriate examples]

(b) *Scenario analysis*

Scenario analysis varies the inputs *simultaneously* to give outputs under different sets of plausible future conditions. [1]

The scenarios are decided upon by:

- looking at historical events, *ie* sets of assumptions based on past data [½]
- generating ideas for possibilities by asking trustees, actuarial consultants, investment analysts or company representatives. [½]

It is often the case that negative scenarios are focussed on, but positive scenarios should also be examined. [½]

No probability of occurrence is assigned to each of the possibilities. [½]

Examples of scenarios:

- Use the assumptions from the time of the previous valuation. [½]
- Pessimistic financial assumptions, *ie* low interest rates, coupled with soaring inflation and salary growth. [½]
- Pessimistic demographic assumptions, *ie* rapidly improving mortality, and increasing numbers of members with dependants entitled to benefits, *eg* a cure for cancer is discovered. [½]
- Optimistic financial assumptions, *ie* economic growth with controlled inflation, giving a high interest rate assumption, and low inflation and salary growth. [½]
- Optimistic demographic assumptions, *ie* high mortality rates and a low proportion of members with dependants, *eg* global flu pandemic. [½]
- Climate change scenario, *eg* high inflation, increased market volatility, increase in credit spreads, increased interest rates, falls in equity returns and property valued, possible changes to mortality rates depending on where scheme is based. [½]
- Statutory basis – may need to consider the scenarios set down in legislation. [½]
- Buy-out basis, *ie* calculate the liabilities assuming that the scheme was closed and its liabilities were transferred to an insurance company. This would mean using assumptions in keeping with those adopted by insurance companies when taking on a pension scheme in this way, and also including some allowance for the insurer's profit and expenses. [½]

[Maximum 2 for appropriate examples]

(c) *Stress testing*

Stress testing is similar to scenario or sensitivity testing but it focuses on *extreme* scenarios or very large changes in input assumptions. [1]

There are two main categories of stress tests:

1. 'top down' stress-scenario tests [½]

2. ‘bottom up’ stress-variable tests, where, instead of looking at a particular scenario and varying all risk factors in a mutually-consistent fashion, the effect of a significant adverse change in a crucial factor (or a narrow range of crucial factors) is considered. [½]

Examples:

- Making a very high (or very low) inflation assumption (*eg* 15% *pa* or 0%) and increasing (reducing) other variables – *eg* interest rates and the salary growth assumption – in a mutually consistent way. [½]
- Sudden improvement in mortality (*eg* to CMI2018 – 10 years) leading to pensions being payable for longer, with a corresponding reduction in the assumption on the existence of dependants. [½]

[Maximum 1 for appropriate examples]

[Maximum 11]

2.3 Generating multivariate normal random variables

Cholesky decomposition

Cholesky decomposition is a method of decomposing a matrix, which can then be used to generate a set of correlated normal random variables from a set of independent standard normal random variables.

Positive definite matrices are always invertible and can be written in the form:

$$\mathbf{M} = \mathbf{C}\mathbf{C}'$$

for some lower triangular matrix $\mathbf{C}$ with positive diagonal entries. A lower triangular matrix is a matrix of the form:

$$\begin{pmatrix} c_{1,1} & 0 & 0 & \cdots & 0 \\ c_{2,1} & c_{2,2} & 0 & \cdots & 0 \\ c_{3,1} & c_{3,2} & \ddots & & \vdots \\ \vdots & \vdots & & & 0 \\ c_{N,1} & c_{N,2} & \cdots & & c_{N,N} \end{pmatrix}$$

The product $\mathbf{CC}'$ is called the *Cholesky decomposition* of $\mathbf{M}$. The matrix $\mathbf{C}$ is known as the *Cholesky factor* and can also be denoted $\mathbf{M}^{1/2}$.

To generate a vector $\mathbf{X}$ with distribution $N_N(\boldsymbol{\alpha}, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma}$ is the covariance matrix (a positive definite matrix), the following algorithm can be used.

Step 1

Perform a Cholesky decomposition of $\boldsymbol{\Sigma}$ to obtain the Cholesky factor $\mathbf{C}$.

Step 2

Generate a vector $\mathbf{Z} = (Z_1, \ldots\ldots, Z_N)'$ of independent standard normal random variables.

You could do this using the Polar method or the Box-Muller method that were studied in earlier subjects. The algorithms for these methods are given in the *Tables*.

Step 3

Set $\mathbf{X} = \boldsymbol{\alpha} + \mathbf{CZ}$.

Question

Perform the above algorithm for when $N = 2$ and $\boldsymbol{\Sigma} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ and $\boldsymbol{\alpha} = \mathbf{0}$

Solution

If $\mathbf{\Sigma} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ then $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix}$

Let $\mathbf{X} = \boldsymbol{\alpha} + \mathbf{CZ} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} Z_1 \\ \rho Z_1 + \sqrt{1-\rho^2}\, Z_2 \end{pmatrix}$

So, if $\mathbf{Z}$ is a column vector of independent standard normal random variables then $\mathbf{X}$ is a column vector of *correlated* normal random variables with mean 0 and correlation coefficient ρ.

Principal component analysis (PCA)

Principal component analysis, or *eigenvalue decomposition*, breaks down each variable's divergence from its mean into a weighted average of independent volatility factors. (In this sense it is similar to factor-based or multi-factor modelling techniques that were discussed in Module 15.)

The theory of *spectral decomposition* states that, for any (square, $N \times N$) covariance matrix $\mathbf{\Sigma}$ there exists a decomposition $\mathbf{\Sigma} = \mathbf{V \Lambda V}'$ where $\mathbf{V}$ is an ($N \times N$) orthogonal matrix that consists of the (unit-length, column) eigenvectors of $\mathbf{\Sigma}$, and $\mathbf{\Lambda}$ is a diagonal matrix containing the eigenvalues of $\mathbf{\Sigma}$.

For example, if $\mathbf{v}_2$ is the *second* eigenvector, then the second eigenvalue is:

$$\lambda_2 = \mathbf{v}_2' \, \mathbf{\Sigma v}_2 = \begin{pmatrix} v_{1,2} \\ v_{2,2} \\ \vdots \\ v_{N,2} \end{pmatrix}' \begin{pmatrix} \sigma_{X_1,X_1} & \sigma_{X_1,X_2} & \cdots & \sigma_{X_1,X_N} \\ \sigma_{X_2,X_1} & \sigma_{X_2,X_2} & \cdots & \sigma_{X_2,X_N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{X_N,X_1} & \sigma_{X_N,X_2} & \cdots & \sigma_{X_N,X_N} \end{pmatrix} \begin{pmatrix} v_{1,2} \\ v_{2,2} \\ \vdots \\ v_{N,2} \end{pmatrix}$$

Each pair of corresponding (unit-length) eigenvectors and eigenvalues may be described as a *principal component*.

For matrices of a large dimension, *eg* $N > 3$, the principal components can be calculated iteratively using the *power method*. However, for matrices of a smaller dimension, *eg* $N = 2, 3$, we can use the following approach (from Subject CS1):

The eigenvalues of a matrix, $\mathbf{\Sigma}$, are the values, λ, such that $\det(\mathbf{\Sigma} - \lambda \mathbf{I}) = 0$ where $\mathbf{I}$ is the identity matrix. The corresponding eigenvector, $\mathbf{v}$, of an eigenvalue, λ, satisfies the equation $\mathbf{\Sigma v} = \lambda \mathbf{v}$ or equivalently $(\mathbf{\Sigma} - \lambda \mathbf{I})\mathbf{v} = 0$. An eigenvector is a vector that a matrix stretches or shrinks. The eigenvalue is the scalar.

Having obtained the principal components, we can then define:

$$\mathbf{L} = \mathbf{\Lambda}^{1/2} = \begin{pmatrix} \sqrt{\Delta_1} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{\Delta_2} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \sqrt{\Delta_N} \end{pmatrix}$$

ie the diagonal matrix containing the square roots of the eigenvalues.

We can then derive $\mathbf{\Sigma}^{1/2} = (\mathbf{V\Lambda V}')^{1/2} = \mathbf{V\Lambda}^{1/2} = \mathbf{VL}$, and use this to generate correlated normal random variables using the formula in Step 3 of the Cholesky decomposition method (described earlier in this section).

Question

Outline the method of simulation of a vector of correlated normal random variables that uses the principal components (developed above), and comment upon how the dimensionality of the problem might be reduced.

Solution

The method of simulation starts by generating a vector $\mathbf{Z} = \left(Z_1, \ldots., Z_N\right)'$ of independent standard normal variables. $\mathbf{X} = \boldsymbol{\alpha} + \mathbf{VLZ}$ is then a vector of correlated normal random variables.

The resulting vector will be affected by the precise method chosen to generate the principal components. For example, $\boldsymbol{\alpha}$, $\mathbf{L}$ and $\mathbf{V}$ may or may not be derived from the sample means and the sample covariance matrix.

The influence of the principal components on the result decreases, *eg* the second principal component has less influence than the first. In some applications this means that a large proportion of the variability can be simulated by a smaller proportion of the total number of principal components. In this case, fewer eigenvalues and eigenvectors are used, and fewer independent random variables need to be generated – thereby reducing the dimensionality of the problem.

2.4 Multivariate normal mean-variance mixture distributions

Let:

- $\mathbf{Z}$ be a column vector, the k elements of which are standard normal random variables
- W be some strictly positive random variable that is independent of $\mathbf{Z}$
- $\mathbf{m}(W)$ be a function of W resulting in a column vector of N elements
- $\mathbf{B}$ be an Nxk matrix of co-scale parameters such that $\mathbf{BB}'$ is in the form of a covariance matrix.

Then $\mathbf{X}$ is said to have a multivariate normal mean-variance mixture distribution if $\mathbf{X} = \mathbf{m}(W) + \sqrt{W}\mathbf{BZ}$. In which case we have: $\mathbf{X}\big|W = w \sim N_N\left(\mathbf{m}(w), w\mathbf{BB}'\right)$.

Special cases of this are:

- The *generalised hyperbolic distribution* where $\mathbf{m}(W) = \boldsymbol{\alpha} + \boldsymbol{\delta}W$ and W has a generalised inverse Gaussian distribution
- The *multivariate t-distribution* where $\mathbf{m}(W) = \boldsymbol{\alpha}$ and γ / W is chi-square with γ degrees of freedom.
- The *skewed t-distribution* where $\mathbf{m}(W) = \mu + \delta W$ (δ is the skewness parameter) and γ / W is chi-square with γ degrees of freedom.

2.5 Multivariate *t*-distribution

An *N*-dimensional *multivariate t-distribution* is described by parameters for location ($\boldsymbol{\alpha}$), scale ($\mathbf{B}$) and shape (degrees of freedom, γ). Notation might therefore be: $\mathbf{X} \sim t_N\left(\gamma, \boldsymbol{\alpha}, \mathbf{B}\right)$.

A contour plot of the bivariate *t*-distribution appears below.

Module 16 Summary – Statistical distributions

Univariate discrete distributions

Binomial distribution: $Bin(n,p)$

- models the number of successes in *n* independent trials where *p* is the probability of success

Negative binomial distribution: $NBin(r,p)$

- models the number of trials needed until there have been *r* successes
- if $r=1$ the distribution is known as the *geometric distribution*

Poisson distribution: $Poi(\lambda)$

- models the number of independent events occurring in a specified time period
- used as an approximation to the binomial distribution for small *p*

Univariate continuous distributions (unbounded)

Normal distribution: $N(\mu,\sigma^2)$

- mathematically tractable distribution (easy to parameterise and use), useful when little is known about the data
- used as an approximation to the binomial and Poisson distributions when the sample size is large
- used to model the error terms in a random walk
- symmetrical and mesokurtic
- by the Central Limit Theorem (CLT), the distribution of the average, $\overline{X}$, of a large sample of *iid* random variables with finite mean, μ, and finite variance, σ^2, is $N\left(\mu,\frac{\sigma^2}{T}\right)$
- tests for normality include *QQ plots* and the Jarque-Bera test
- the test statistic, $Z=\frac{X-\mu}{\sigma}\sim N(0,1)$, is used to determine, whether a single observation, *X*, is significantly different from μ where σ is known
- the test statistic, $Z=\frac{\overline{X}-\mu}{\sigma/\sqrt{T}}\sim N(0,1)$, is used to determine, whether the sample mean, $\overline{X}$, is significantly different from μ, where T is the number of observations and σ is known

Normal mean-variance mixture distribution: $N\left(m(W), W\beta^2\right)$

- introduces randomness into the mean and variance
- $X = m(W) + \sqrt{W}\beta Z$ where W is a strictly positive random variable, $m(W)$ is a function of W, $Z \sim N(0,1)$ is independent of W, and β is a scale parameter

(Generalised student's) t-distribution: $t(\gamma, \alpha, \beta)$

- used to model symmetric data sets where the tails are fatter than implied by a normal distribution (leptokurtic) – important distribution for modelling risks
- can be derived as a normal mean-variance mixture distribution
- for finite values of γ the tail of the distribution follows the power law – the probability of an event falling approximately in inverse proportion to the size of the event raised to the power of $\gamma + 1$
- as $\gamma \to \infty$, the distribution tends to the $N(\alpha, \beta)$ distribution
- the test statistic, $Z = \dfrac{X - \mu}{s} \sim t_{T-1}$, is used to determine whether a single observation, *X*, is significantly different to μ, where *T* is the number of observations and *s* is the sample standard deviation
- the test statistic, $Z = \dfrac{\overline{X} - \mu}{s/\sqrt{T}} \sim t_{T-1}$, is used to determine, whether the sample mean, $\overline{X}$, is significantly different from μ, where T is the number of observations and *s* is the sample standard deviation

Skewed t-distribution: $t(\gamma, \alpha, \beta, \delta)$

- positively skewed if $\delta > 0$, negatively skewed if $\delta < 0$ and symmetric if $\delta = 0$

Key advantages of the Merton model are that:

- It is mathematically tractable, using results from Black-Scholes option pricing.
- It results in an intuitive, economic explanation for the probability of default, since it is based on the capital structure of a company and changes in the value of this company.
- It allows us to estimate an appropriate credit spread for a bond, even when the bond is unquoted and/or unrated.

However, there are various drawbacks and assumptions involved in the structural approach. We assume:

- markets are frictionless (*ie* no transaction costs) with continuous trading
- the risk-free rate is deterministic and constant for borrowers and lenders
- X_t follows a log-normal random walk (geometric Brownian motion) with a fixed rate of growth and fixed volatility (*ie* independent of the company's financial structure, *eg* level of gearing) – an unrealistic assumption
- X_t is an observable traded security – this assumption is rarely correct
- the bond is a zero-coupon bond with only one default opportunity
- default results in liquidation – however, default can mean a variety of things other than liquidation in real life.

The above approach gives an equation that can be solved only when we can observe X_t and σ_X, which is normally impossible.

Since an accurate estimate of σ_X is needed, the Merton model is more appropriate for larger companies with frequently-traded stock.

A further limitation of the approach is that the results can be affected significantly by changes in market sentiment in the absence of any real changes in a counterparty's prospects.

3.2 The KMV model

The *KMV model* uses the concept introduced by Merton that a company will default at the first instance that X_t falls below *B* (or some other level $\tilde{B}$ which is determined by looking at the term structure of all the company's liabilities, *eg* $\tilde{B}$ is often taken as the liabilities falling within one year).

The KMV model uses the concept of the *distance to default* (DD), which is the number of standard deviations that the company's assets have to fall in value before they breach the threshold $\tilde{B}$.

$$DD_0 = \frac{X_0 - \tilde{B}}{\sigma_X X_0}$$

Using empirical data on company defaults and how these defaults link with the DD, the model is used to estimate the likelihood of default for any given company over the coming year.

Question

Outline three key advantages of the KMV model over the Merton approach.

Solution

Three advantages of the KMV model over the Merton model are:

1. Coupon-paying bonds can be modelled.
2. More complex liability structures can be accommodated as the system uses the average coupon and the overall gearing level (rather than having to assume a single zero-coupon bond).
3. X_t is not assumed to be observable and is derived from the value of the company's equity shares.

Although not asked for by the question, we note here that the results of the KMV model can (like the Merton model) be affected significantly by changes in market sentiment in the absence of any real changes in a counterparty's prospects.

4 Credit-migration models

For longer-term exposure to counterparties (over more than one year), credit-migration models estimate how the credit rating might change over time. Credit migration models are a type of reduced form model. They were introduced in Subject CM2, for example, the two-state model and the Jarrow-Lando-Turnbull (JLT) model.

Estimating the probability of default in each future year

The modelling process generally has three steps:

1. Historical data is used to determine the probability that a company rated (say) A at the start of a year will be rated A–, A, A+, or indeed any other rating, at the end of a year. These probabilities are recorded in *rating transition probability matrices*.

2. These matrices are applied (repeatedly) to a counterparty's current rating to estimate the likelihood of each possible rating in each future year.

3. Then using the probability of default for a company of a given rating, the model estimates the chance of default in each future year.

Question

Outline one technical advantage and three technical disadvantages of this approach to modelling the probability of default in each future year.

Solution

A technical advantage of the credit migration approach is that volatile equity markets should not overly impact the results – empirically, this is often the case for the actual default probability of a particular firm.

Technical disadvantages include the following questionable assumptions:

1. The likelihood of default is determined solely by the company's credit rating.

2. This credit rating reflects the average company's default likelihood 'throughout the business cycle' (rather than reflecting the default chance in the current economic environment).

3. The credit migration process follows a time-homogeneous Markov chain, and hence there is an assumption that history will repeat itself.

The time-homogeneity assumption has been criticised using empirical evidence, which indicates that default rates for companies with a particular credit rating vary over time. It also appears unintuitive, as a recently downgraded company is more likely to be downgraded again than a company that has been at that rating for a long time.

A simpler form of credit-migration model assumes that credit migration follows a martingale process. However, at least for the highest credit rating, it is incorrect to assume that the expected rating one time-period on is the same as in the current period. As a consequence, over multi-year periods the results can only be regarded as very rough estimates for default.

CreditMetrics (single bond)

CreditMetrics (described in more detail below) estimates of the value of a bond in one year's time for each of its possible future ratings and deduces the bond's expected future value. Combining this information with the transition probabilities produces an estimate of the variance of the bond's value in one year's time.

Advantages and disadvantages of bottom-up models

Question

Outline the argued main advantage of bottom-up models, and three limitations of these approaches.

Solution

There is increasing preference for bottom-up models to be used as they give a more robust picture of a company's overall risk profile.

However, these models still have limitations:

1. It is difficult to break down reported aggregate losses into their constituent components (*eg* in order to apply non-life reserving techniques to the components, each of which has a different risk of loss).

2. There may be little robust internal historic data, especially for low probability and high- (but uncertainly high-) impact events.

3. Differences between companies mean that application of external data (to supplement limited internal data) is difficult.

2.2 Scenario Analysis

In Module 15, *scenario analysis* was discussed primarily within the context of mathematical modelling. In this section we consider how the process of scenario analysis might encompass the evaluation of risks which are harder to model mathematically, *eg* low-probability high-impact operational risks such as fraud.

An advantage of this broader process is that it can reflect the potential linkages between operational and other risks within each scenario.

Question

Outline the steps involved in a process of applying scenario analysis to an assessment of a set of non-financial (*eg* operational) risk exposures.

Solution

Scenario analysis involves a number of steps:

- Risk exposures need to be grouped into broad categories – all risks involving financial fraud; all risks involving systems errors, for example. This step is likely to involve input from a wide range of senior individuals in the organisation.

- For each group of risks, a plausible adverse scenario is developed. The scenario needs to plausible, otherwise it will not be possible to determine the consequences of the risk event. The scenario is deemed to be representative of all risk in the group.

- For each scenario, the consequences of the risk event occurring are assessed. Again this is likely to involve input from senior staff / experts.
  - The financial consequences include redress paid to those affected; the cost of correcting systems and records; regulatory fees and fines; opportunity costs while any changes are made; *etc*. In practice the mid-point of a range of possible values is usually taken.
  - The non-financial consequences may take the form of operational strain and/or regulatory interest.
- The total costs calculated are taken as the financial cost of all risks represented by the chosen scenario.

Note: The above is largely Core Reading from Subject CP1.

Although arguably a more subjective approach than pure quantitative analysis, scenario analysis has benefits including:

- capturing the opinions, concerns and experience of risk managers
- not relying heavily on the availability / accuracy / relevance of historic data
- providing an opportunity to identify hard to predict, high-impact events (so called *black swan* events)
- identifying and improving understanding of cause and effect relationships
- reducing risk-reward arbitrage opportunities.

2.3 Factor-based models

As an alternative to the above approaches, a simpler approach might be adopted. For example, where statistical analysis is not feasible, perhaps due to a lack of data, it may be assumed that losses are related to the volume of transactions (by number or value). In such a case, an assessment of operational risk may be obtained by applying a weighting to the actual or expected volume of transactions.

Question

Outline a key disadvantage of such factor-based models in general.

Solution

Operational risk exposure may *not* be proportional to business volume, *ie* volume is not necessarily a suitable proxy for risk exposure.

Specific examples of such factor-based models include the Basel basic indicator and standardised approaches, which are described below.

Business units may be incentivised to improve operational risk management if operational risk is included in the allocation of economic capital (hence reducing their capital charges).

1.8 Bias

The problem of bias and how to avoid it was discussed in Module 13.

In summary, to avoid bias:

- checks and balances should be built into the system
- assessments should be subjected to competent and genuinely independent checking
- consider introducing an 'optimism bias' into the appraisal of capital projects
- educate people about the problem of unintentional bias.

1.9 Process risk (change management)

The introduction of changes into business processes or IT systems introduces the risk to the business that the new processes or systems may fail or be poorly implemented.

Question

State how such process risks can be managed

Solution

Techniques to manage process risks include:

- undertaking pilot studies
- precise definition of the requirements of any new solution to best meet the needs of the whole enterprise
- designing systems that can be easily maintained, enhanced and upgraded
- careful deployment of the new systems with user education.

The stress testing of any new process or system should be done both in isolation of and within the larger structure into which it is to be placed.

Having been introduced, processes should be reviewed regularly for effectiveness.

1.10 Model risk and Data risk

Model risk was discussed in Module 21.

Question

Suggest ways in which model risk and data risk can be managed.

Solution

To manage model risk, it is important to:

- ensure a robust process around the choice of model
- have documented processes for the model and assumptions
- have clear audit trails and change-management routines
- test the model thoroughly before use
- maintain and develop the model over time, with regular reviews
- ensure staff are adequately trained and that there are clear accountabilities
- understand the key drivers / assumptions in the model …

  … and subject the model to tests of parameter uncertainty
- use models only for their intended purpose
- appreciate the limitations of the model
- avoid overly complex models (principle of parsimony)
- ensure workings and results are easy to communicate and appreciate …

  … and capable of independent verification for reasonableness

To manage data risk, it is important to:

- limit what can be entered to what is valid, *eg* range checks
- check data entry (spot checks, consistency checks, reasonableness checks)
- re-check data on transfer and, in particular, de-duplicate
- ensure the data is credible (sufficient quantity) …

  … and relevant to the purpose (especially if using external data)
- carry out regular backups of data
- ensured that data is stored securely
- ensure staff are adequately trained, *eg* in data protection, handling big data.

30

Capital management

Syllabus objectives

7.1 Demonstrate an understanding of capital calculations.

7.1.1 Describe the concept of economic measures of value and capital, and their uses in corporate decision-making processes.

7.1.2 Evaluate different risk measures and capital assessment approaches.

7.1.3 Demonstrate the ability to develop a capital model for a representative financial firm.

7.2 Propose techniques for allocating capital across an organisation.

0 Introduction

The concept of capital was introduced in Module 3. At a basic level, capital means wealth or financial resources. However, there are many different types of capital, which we introduce here and then expand on in the main body of the module. The definitions of capital used in this module are not definitive and, as with risk, it is important to recognise that different organisations will have different taxonomies.

Question

Outline the main reasons why a financial organisation (such as an insurance company or a bank) requires capital.

Solution

A financial organisation requires capital to:

- cover unexpected losses arising from exposure to risks associated with its existing assets and liabilities (*risk capital*)
- support its ongoing business strategy (*working capital*), for example to:
  - fund the writing of new business, *ie* new business strain
  - demonstrate financial strength (*eg* by achieving a particular target credit rating) in order to attract new business
  - fund overheads and development costs, *eg* new IT systems, premises, development of a new product or sales channel
  - fund a merger or acquisition.

It is important to distinguish between:

- the capital required by an organisation to withstand risks and support its business strategy (*required capital*)
- the capital available to meet this requirement (*available capital*).

In this module we focus in particular on required capital, how to assess it and how to allocate it around an organisation. There are various different assessments of required capital:

- an organisation's *own* assessment of the required capital (*economic capital*)
- a *third party's* assessment of the required capital for example:
  - by regulators (*regulatory capital*) according to some prescribed formula or calculation approach, *eg* the Basel formula or Solvency II SCR approach
  - by rating agencies (*rating agency capital*), *eg* to achieve a particular credit rating.

This module considers both the development of capital models and issues relating to the allocation of capital across separate parts or segments of an organisation.

The reading material for this module focuses on the concept of economic capital, but the principles (modelling techniques and capital allocation approaches) can apply similarly to regulatory required capital.

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so that you can easily remove and use the task list.*

1 Capital

1.1 Definitions of capital

The capital that an organisation has access to acts as a buffer in the event the organisation faces risks that impact its balance sheet. Allowing for differences between various valuation bases, it corresponds to an item that might be known by a variety of terms, including as *Net Asset Value*, *Shareholders' Funds*, *Free Assets*, *Free Reserves*, and *Own Funds*. Whichever name it is known by, it effectively corresponds to capital that 'belongs' to the stakeholders who provide capital, and is not expected to be needed if events turn out according to the central estimates of the organisation.

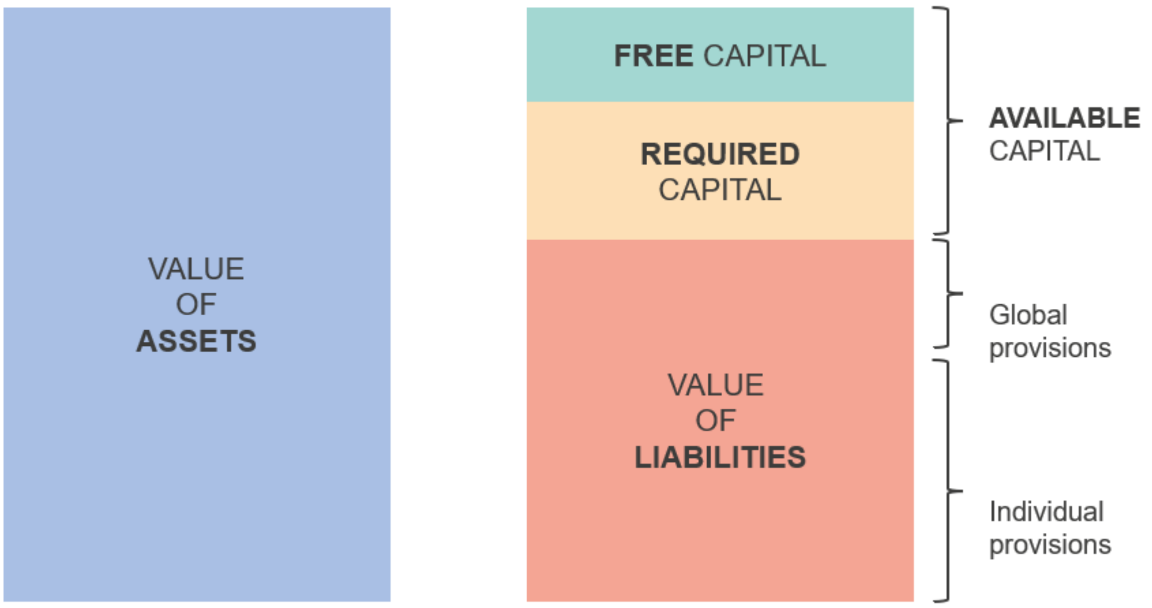
This capital that an organisation has access to (the value of the assets less the value of the liabilities) is also known as *available capital*. It comprises items such as ordinary share capital, retained earnings, revaluation and other reserves, and subordinated debt. It may help to think back to Subject CB1, and the makeup of the capital section on the balance sheet.

Unless the organisation is under material financial distress, the capital available is usually significantly above the regulatory and economic capital that might be calculated by an economic capital model.

Here, a distinction is being made between capital that is *available* to an organisation, and capital that the organisation is *required* to hold, for example as assessed for regulatory purposes (regulatory capital) or internally by the organisation (economic capital).

Any available capital in excess of the required capital is generally referred to as *free capital* or *excess capital*.

The following diagram illustrates how, for an insurance company, the different categories of capital link in with each other.



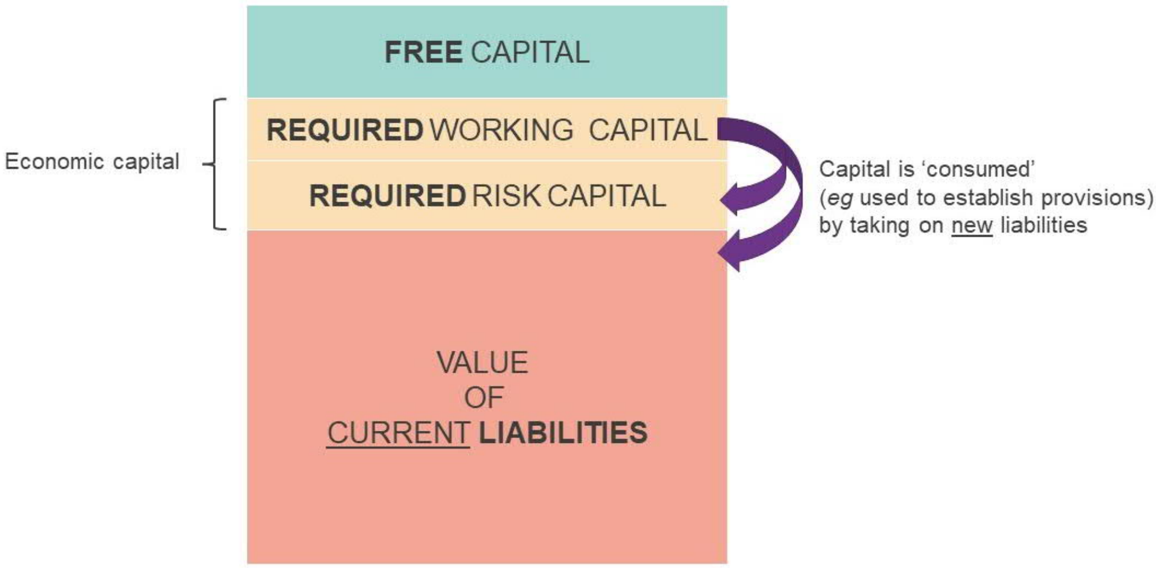
1.2 Required capital

As stated in the introduction to this chapter, the requirement to hold capital may be driven:

- internally by the organisation (*economic capital*)
- by the regulator (*regulatory capital*)
- by rating agencies (*rating agency capital*).

These assessments of required capital may be different. Typically they will all involve an assessment of risk capital, *ie* the capital required to cover unexpected losses arising from exposure to risks associated with its existing assets and liabilities. However, the various assessments may reflect different risk types, different risk measures, different time horizons, different ways of aggregating risks, different bases for valuing assets and liabilities, and different modelling approaches.

Additionally, the economic capital assessment may encompass capital needed to support ongoing business objectives over the time horizon chosen. For example, such business objectives might encompass writing new business, developing a new product and/or maintaining a certain credit rating. In other words, economic capital may also include an element of *working capital*.



The most onerous of the capital assessments (*eg* economic, regulatory) will vary from one organisation to another. For some organisations, the economic capital assessment will be higher than the regulatory capital assessment. For other organisations, the reverse may be true. It depends on many factors such as the nature of the existing business, the organisation's risk tolerance and its plans for the future.

1.3 Free capital

The Board will need to decide how much of the free capital should be retained (*ie* to provide an additional buffer beyond that accommodated by the current assessment of required capital), and how much should be returned to the shareholders (*eg* by way of dividends and/or share buybacks).

1.4 Required risk capital

In the remainder of Section 1, and in Section 2, we are mainly concerned with required risk capital, *ie* the capital required to cover unexpected losses arising from exposure to risks associated with an organisation's existing assets and liabilities. This risk capital may be calculated as part of an economic capital assessment or as part of a regulatory capital assessment. For brevity, we shall refer to this simply as 'capital' from this point onwards.

Although there are many different definitions of capital in use, there are some common threads:

- capital should provide sufficient surplus to cover *adverse outcomes*
- with a given level of *risk tolerance*
- over a specified *time horizon*.

Question

State three definitions of capital that result from the three main interpretations of the meaning of 'adverse outcomes' (above).

Solution

The three main interpretations of 'adverse outcomes' result in the following definitions of capital:

1. the surplus needed to cover all potential outgoings, reductions in assets and/or increases in a company's liabilities at a given level of risk tolerance over a specified time horizon
2. the surplus needed to maintain a given level of solvency at a given level of risk tolerance over a specified time horizon
3. the excess of the value of the assets over the value of the liabilities at a given level of risk tolerance at a specified time horizon.

Unlike the first two definitions, the third definition above focuses on the values of the assets and liabilities *at* the specified time horizon – rather than the funding position (cashflow or surplus) of the company *throughout* the time period concerned.

Question

State three metrics that might be used to set an appropriate level of risk tolerance.

Solution

The appropriate risk tolerance level might be set with reference to:

1. a certain *percentile* of the loss distribution
2. extreme loss *values*
3. the possibility of some key *indicator* (*eg* credit rating) falling outside an acceptable level.

Other definitions

There may be other ways to assess capital requirements within an organisation, such as regulatory standard formulae (or other prescribed calculations) and rating agency factor-based models. Where relevant, these are also an important part of the capital management process.

These are discussed later in this module.

1.5 Capital models

Capital models (CMs) can be used for a number of purposes within an organisation, such as regulatory capital setting or considering economic capital requirements.

A model used for the latter might be referred to as an economic capital model (ECM).

Best-practice models are also able to allocate the capital across the company. (Allocation of capital is discussed in Section 4 below.)

Generic capital models

A *generic capital model* may be used by capital providers and regulators to gain a *consistent* assessment of capital requirement across different firms.

Such models can be very simple in application, *eg* factor-table approaches determine capital as a multiple of business volume. However, generic models are becoming more complex due to:

- the increased sophistication of risk management practices adopted internally at companies
- previous models failing to deal properly with all risks
- increasing pressure on companies to optimise their capital resources.

Internal capital models

An *internal capital model* is used to simulate a *company-specific* view of the capital needed. Internal CMs should aim to cover:

- all risks faced by a company ...
- … in a consistent way ...
- ... allowing for the interaction between the various risks.

Internal CMs can also be used to calculate *regulatory* capital (as an alternative to using a *generic* CM). However internal CMs used for this purpose require detailed scrutiny by the regulator.

Any model is a subjective tool and the results should be used with care. However, careful use of an internal capital model can help to provide a sound basis for the development of a risk management strategy targeting the agreed risk objectives (*eg* reducing earnings volatility).

3 Capital as a management tool

As mentioned in Module 9, *risk capacity*, which represents an upper bound for the size of the organisation, may be determined by the amount of available capital. Risk limits relate to risk capacity, in that they act to ensure the organisation's requirement for capital does not exceed the capital available to it.

In practice, managers responsible for the various parts of an organisation might be allocated a finite subset of required capital. Such an allocation would act as an upper bound within which they must operate. Some of that allocation will relate to existing business – it is required to act as a buffer against adverse outcomes associated with these existing liabilities. The remainder, if any, of the allocation will provide headroom for expansion / business growth. So, for example, if the Board of an insurance company wishes to limit (or grow) the amount of new business written within a particular line of business, then they might act to limit (or increase) the allocation to that line.

For example:

Business line	**A**	**B**	**C**	**TOTAL**
Strategy	Limited growth	Contract / Run off	Expand	
Allocation of required risk capital (existing business)	100	50	25	175
Allocation of required working capital (for growth)	20	–10	25	35
TOTAL ALLOCATION	120	40	50	210

Each part of the business will 'use' some, but not necessarily all, of their capital allocation. Allocated capital which is left 'unused' will act as a drag on performance – an opportunity cost – and so should be monitored. Conversely, the performance of 'used' allocated capital (*ie* return on capital) should be measured, so as to inform strategic decision making. For example, if a line of business is underperforming then it might be closed to new business or, conversely, if it is performing well, it might be awarded a greater allocation of capital (leading to higher risk limits) so as to facilitate / encourage growth.

The next section of this chapter considers how return on allocated capital might be measured. The subsequent section considers how the allocation of capital might be made. After that, in the final section of this chapter, an example is presented to act as the basis for a discussion of issues surrounding how capital might be used as a management tool.

4 Risk optimisation

As mentioned in Module 26, the objective of risk management is to optimise the balance between risk and return – where optimality is judged by reference to risk appetite. Risk management is not simply about minimising risk.

Understanding return on capital is one way of ensuring that an organisation is putting its limited capital resources to the best use. The main measure is the risk-adjusted return on capital (RAROC).

Risk-adjusted return on capital (RAROC)

$$RAROC = \frac{risk\text{-}adjusted\ return}{capital}$$

RAROC:

- can be calculated for an organisation as a whole
- can be calculated for separate parts of the organisation and/or projects, and used to compare them, *ie* to identify activities that are creating or destroying shareholder value (by comparing RAROC versus cost of capital)
- can be based on actual or expected return and actual or expected capital.

Note that there is no single definition of either 'return' or 'capital', so RAROC is not well defined. However, an internal assessment of RAROC might reference economic capital in the denominator.

Question

Outline how the RAROC might be determined, by expanding the numerator in the above formula to show the possible components of its calculation.

Solution

The risk-adjusted return on capital (RAROC) may be calculated as:

$$RAROC = \frac{expected\ revenues - costs - losses + return\ on\ capital - taxes}{capital}$$

Economic income created (EIC)

Unlike RAROC, *economic income created* (EIC) captures the *quantity* of return generated by a unit of activity (*ie* it is a monetary amount rather than a percentage):

$$EIC = (RAROC - hurdle\ rate) \times capital$$

As EIC is a monetary amount, it can be used to encourage marginal growth opportunities, *ie* those activities that do add value, yet may not meet RAROC targets.

Question

Describe the risks that are likely to be faced by Company A in relation to its legacy defined benefit pension fund.

Solution

Risks to Company A in relation to the legacy defined benefit scheme include:

- funding risk – the risk that Company A needs to pay higher contributions to the scheme than expected to make good a shortfall, due to, for example:
  - longevity risk, resulting in benefits being paid for longer than expected
  - inflation risk, resulting in higher pension payments than expected
  - investment risk, *eg* from worse than expected investment returns, a market crash, mismatching of assets and liabilities, failure to recognise climate change risk.
- covenant risk, *ie* a poor assessment of Company A's current and ongoing financial ability to support the scheme, exposing Company A to the risk of needing to pay higher contributions to improve the security of the scheme
- liquidity risk, *eg* insufficient liquid assets being held to meet funding obligations as they fall due and therefore assets being realised at inopportune times …

  … and at a lower value than would otherwise be the case
- operational risks such as:
  - regulatory risk, from changes in regulation, *eg* gender equalisation of benefits
  - governance risks, *eg* impact of trustee bias or mismanagement of trustee conflicts of interest
  - administrative risks *eg* administrative errors leading to unexpected costs
  - cyber risks, *eg* a data or security breach leading to fines, reputational damage.

Under Solvency II, insurance companies are required to consider the risk that a pension scheme poses in the assessment of the SCR. In particular consideration should be given to the impact that adverse events may have on the scheme, and hence on the sponsoring company, as well as any commitment to make good a shortfall in the scheme.

The following questions need to be considered as part of any work to consider capital allocation. Some need to be considered before the work is done, while others can only be considered during the work. All questions should be addressed, as far as possible, in advance of any risk events occurring.

Questions relating to the pension fund

- **How should the risk of the pension fund be recognised?**

  For example, should the risk be measured as a multiple of any shortfall in the pension fund, or should the risk be measured based on the additional funding requirements needed under adverse stresses?

- **Should a proportion of capital be carved out and notionally allocated to the fund, recognising that it is effectively expected to make a zero return?**

  Allocating some of the available capital to the pension fund would reduce the free capital available to invest in projects that would be expected to make a non-zero return.

- **Does it provide a better shareholder return in the long run if Company A suspends dividends for a while so as to make the pension fund fully funded?**

  Suspending dividends now will reduce shareholders' short-term income yield. However, by reducing the pension fund's deficit (sooner), total required capital will reduce (sooner) and so RAROC will increase (sooner). So, the deferral of dividends may, on balance, be beneficial to medium- / longer-term investors.

- **If that choice is made, what will the attitude of those shareholders be to the failure to provide them with income in the short run?**

  Will shareholders (especially those with short-term investment horizons) vote against the reduction? Will the share price be adversely affected?

Questions relating to the catastrophe excess of loss line

When there is a bad catastrophe year, how should the financial loss be recovered?

- **Should capital be withdrawn from the catastrophe excess of loss line, so suppressing its business volumes?**

  Catastrophe excess of loss lines are sold on the basis of financial strength and reliability. Having lots of capital allocated to the existing business is an indicator of strength (and hence promotes sales). Conversely, withdrawing capital allocated to existing business reduces the perceived strength (and hence suppresses sales). Suppressing business volumes in turn reduces the future exposure to risk, *ie* limits any financial losses.

- **Is it better to allocate *more* capital to that line, in expectation of higher future premiums and therefore higher future returns?**

  By increasing the capital allocation to the catastrophe excess of loss line (*ie* allocation of working capital required for growth), its managers have greater freedom to generate and accept larger business volumes, and hence to achieve a higher subsequent RAROC.

- **Will that be seen as a reward for failure and cause friction between the business lines?**

  Managers of other lines of business, which are achieving a higher RAROC, may well feel that they are more deserving of being allocated more capital required for growth than a line which is suffering underwriting losses.

Questions relating to the household line

- **How should Company A decide on the level of catastrophe excess of loss its household line buys?**

  The level of catastrophe excess of loss reinsurance may be determined by a number of factors such as the cost of the reinsurance *vs* the benefit in terms of the reduction in risk (and hence in terms of reduced volatility in profits and in reduced required capital), the risk tolerance of that line and hence the desired retention level (attachment point).

- **Should it allow the household underwriting team to decide on their own purchase?**

  This might result in them deciding to buy with a low attachment point, to protect their own results in the event of a catastrophe.

  Here 'results' is referring to the household line's underwriting result, which is only one of the performance metrics which should be considered. The lower the attachment point (retention level), the less risk is retained by the household line, and hence the more stable the results are likely to be. However, a lower attachment point means a greater reinsurance cost

- **Or should it set the attachment point centrally based on what is optimal for the company?**

  From the perspective of capital efficiency, one way of seeking to optimise the attachment point would be to maximise expected RAROC. The lower (higher) the attachment point the lower (higher) the risk retained and the lower (higher) the capital allocation to the household line (all other things being equal). That reduces (increases) the denominator in the RAROC metric, however the numerator will also be reduced (increased) by the higher (lower) cost of reinsurance.

  A higher (centrally determined) attachment point might result in the household underwriting team deciding to restrict their business volumes so as to reduce the chances of them failing to meet profit targets as a result of a catastrophe – and so cause an expense strain.

  A higher attachment point means that the household line retains more risk and so there is a greater chance that profits are adversely affected by a catastrophic event. One way of countering this is to take on less risk in the first place, *ie* to restrict business volumes.

  The 'expense' strain referred to is the fact that the household line is less likely to be able to cover its overheads if fewer policies are written.

Questions relating to the motor line

- **What level of capital should be allocated to the motor line?**

  It is weakly correlated with the other lines, so should it receive a very small allocation to recognise that – which will incentivise the motor team to boost their volumes and hence their potential profits and bonuses?

  The weak correlation means that the motor line is acting as a diversifier to the other lines, *ie* reducing the overall risk. If we employed a capital allocation method (*eg* Euler) that recognises the diversifying contribution of each line, this would reduce the amount of capital allocated to the existing business of the motor line.

A reduced capital allocation means that the performance of the motor line, as measured by RAROC, may increase (due to a lower denominator). Equivalently, if the motor line is targeting a particular RAROC, and the capital (denominator) has reduced, then a lower return (numerator) is needed to hit the target ratio. In other words, the motor line can reduce the premiums charged, and this is likely to boost sales volumes.

If the bonuses of the sales team are linked to RAROC, this may incentivise the team to write more business. Additionally, a high RAROC relative to other lines may mean that the motor line's ranking (or priority) within Company A is potentially improved.

Furthermore, a low capital allocation to existing business reduces the 'cost of capital' charge in the premiums, helping to make premiums for the motor line more competitive, which in turn would boost volumes.

- **Or should it receive a much larger allocation to recognise that it runs a significant risk of large claims?**

  This is referring to one of the introductory statement, which says the motor line is exposed to individual large claims.

- **If the allocation is much larger than the allocation typically used in the market, this could force the team to price at a level above what the market can stand, and so significantly reduce their volumes.**

  A higher capital allocation to existing business results in a higher 'cost of capital' charge in the premiums, making premiums for the motor line less competitive.

Questions relating to the asset management line

- **How should the asset management team be incentivised?**

  Although their impact on the economic risk of Company A is small, they have a material contribution to the operational risk.

- **Can the methodology of capital allocation be adapted so as to take account of this contribution?**

  If the capital modelling does not include any allowance for operational risk, then this should be corrected. For example, the required capital for operational risk might be determined as a proportion of invested funds (*ie* a proxy factor-based approach akin to the Basel method). The asset management business' total operational risk capital might be allocated across sub-teams using a pro-rata method, *eg* in proportion to the existing capital allocation.

Module 30 Summary – Capital management

Definitions of capital

It is important to distinguish between:

- the capital required by an organisation to withstand risks and support its business strategy (*required capital*)
- the capital available to meet this requirement (*available capital*).

Financial organisations require capital to:

- cover unexpected losses arising from exposure to risks associated with existing assets and liabilities (*risk capital*)
- support ongoing business strategy (*working capital*), for example to fund the writing of new business, the development of a new product or a merger / acquisition:

Common threads in various definitions of risk capital include:

- capital should provide sufficient surplus to cover adverse outcomes
- with a given level of risk tolerance
- over a specified time horizon.

The requirement to hold capital may be determined internally (economic capital) or externally (regulatory capital, rating-agency capital).

Capital models

A generic capital model may be used by capital providers and regulators to gain a consistent assessment of capital requirement across different firms. Such models can be very simple in application, *eg* factor-table approaches.

An internal capital model (ICM) is used to determine a company-specific view of the capital needed (economic capital). They can help improve management's understanding of the dynamics of the business and thereby improve decision making. ICMs should aim to cover:

- all risks faced by a company
- in a consistent way
- allowing for the interaction between the various risks.

ICMs might be used to calculate regulatory capital (subject to regulatory approval).

There are six stages required in operating a successful capital model:

1. identify purpose
2. identify and rank risks
3. choose the simulation approach for each risk
4. define the risk metrics
5. select the modelling criteria
6. decide on the method of implementation.

Uses of an internal capital model include:

- simulating a company-specific view of the capital needed, *eg* as an alternative to using rating agency / regulatory assessments
- determining company or product risk profile
- capital budgeting
- assessing the impact of strategic decisions
- pricing insurance products
- setting risk tolerances/constraints
- setting investment strategy
- calculating risk-adjusted rate of return on capital
- measuring performance
- determining incentive compensation
- modelling the impact of extreme events
- disaster planning.

Capital management

Effective capital management can increase shareholder value by facilitating better: pricing, reserving, performance management and risk management.

Capital management involves: quantification (*eg* using capital models), allocation and optimisation.

Quantitative elements of the ORSA might be assessed using an ICM.

An insurer's ORSA must include a *continuity analysis* – of its ability to continue in business.

Writing this review from the current perspective also brings into focus another major risk event, one which has unexpectedly dominated the agenda globally – the 2020 Covid-19 pandemic.

1.10 Covid-19

Major risk events tend to be analysed publicly some time after the event, as they are either constrained with a single organisation or emerge as a result of complex interlinkages between organisations which take time to unravel. However, CROs and risk departments often have to react in real time when dealing with smaller risk events as they emerge, and make recommendations based on incomplete and partial information.

The 2020 Covid-19 pandemic illustrates a mixture of both: it is a major risk event, but is having to be dealt with by most, if not all, organisations in real time.

This section was written in December 2020 when the first vaccination programmes were being rolled out in the UK and the US, and the EU had just granted approval for the vaccine. However, the situation is continuing to develop with new strains being identified, and while most lockdowns have been eased, the possibility exists that further lockdowns may be enforced.

Background

During December 2019 a new Coronavirus, SARS-CoV-2, was identified, which results in a disease known as Covid-19. This disease is highly contagious, but has a reasonably long asymptomatic incubation period during which carriers can transmit the disease. It results in painful and disruptive symptoms, which can persist for months. It is deadly to many of those infected, especially those who are elderly or who have underlying health conditions.

By March 2020 the disease had spread widely across Asia, Europe and beyond, and most countries began various programmes of lockdown – effectively banning a substantial amount of economic activity and providing very substantial government support to a very wide range of businesses.

The Response of Risk Managers

Prior to 2020, pandemic risk will have been on many organisations' risk registers. It is most likely to have been analysed as an operational risk scenario – for instance, 'Let us assume that 25% of our staff are off sick', and possibly as a mortality risk scenario – for instance, 'Let us assume that our mortality rates are 10% higher in one year than we expect.'

Pandemic risk was also on government risk registers. It was widely reported in 2020 that pandemic flu was the highest impact risk identified by regular UK government risk civil emergency risk reviews. Being reported is not the same as being believed. Being believed is not the same as doing something about it. It is notable that those countries with recent experience of fast-moving pandemics, notably in parts of Asia, seem to have been much better prepared than countries without that experience. For most of Europe and North America the last similar event was the influenza pandemic of 1918-1920.

This situation highlights two important factors in risk management. First, the pandemic is a classic black swan risk: a risk that is very low probability with very high impacts and which seems obvious in hindsight. It is very difficult to encourage people to think about mitigants to black swan risks.

Black swan risks were defined in Module 15.

Second, even if a Board was convinced that mitigants were required for a pandemic risk, what, on its own, should it do. Put another way, the actions that have been taken by governments worldwide in response to the pandemic were unthinkable in January 2020. Any mitigant at organisational level could be swept away by a government action. It is impossible to know what actions a firm wishing to be well-prepared could take to be confident of being better off as a result.

Perhaps the best strategy is to respond appropriately to the developing situation, which indeed is how most organisations have had to react. Those with risk functions, it is hoped, will have assessed the risks as they have been identified, and fed that intelligence into the decisions made by their organisations. In this section, I have set out the examples of emerging risk considerations. To provide some structure, the following material will use elements of the risk taxonomy implicitly provided in Module 3.

Financial market risks

This section considers market risk, interest rate risk, foreign exchange risk, basis risk, credit risk, counterparty risk, liquidity risk.

In March 2020 the financial markets saw very significantly increased volatility, with very substantial selloffs. During late February and early March both FTSE100 and S&P500 lost about a third of their value. There were also wild variations in both government bond yields and in bond spreads. During this period of volatility there were three main financial market risk questions for financial institutions to address:

(a) What is this asset value and interest rate volatility doing to our current capital levels? Do we need to take urgent action to return them to an acceptable level?

(b) How is the volatility impacting our risk limit system? Are we in breach of any of our risk appetites, for concentration, liquidity, or value? Should we change our risk appetite to accept any breaches as a temporary measure or should we take action to restore exposure so that it is within appetite?

(c) To reflect the emerging situation, will the regulators make changes which affect the capital and risk limits that are permitted? How quickly will the regulators respond?

Since March, most investment indices have rebounded very significantly. CROs have to consider whether this is a signal of confidence in the economic future, or irrational optimism signalling a future correction, perhaps once government support is withdrawn and the true extent of economic scarring becomes clear, and advise their colleagues accordingly. An organisation needs to understand its risks so that it is confident it is resilient to responses at any point including at the two extremes.

Economic risk and strategic risk

For all organisations there was an urgent requirement to understand the potential impact of government economy shut-down measures on their finances, and to work out how best to access government support. Some organisations had no choice but to accept that support as the price of staying afloat in the hope of better times to come. However, it is notable that some organisations who had access to that support either turned it down or later returned it, to mitigate reputational risk.

The Core Reading for Subject SP9 refers to the Core Reading from the following earlier subjects:

- CS1 Core Reading
- CS2 Core Reading
- CM2 Core Reading
- CP1 Core Reading

1 Specific ERM topics

Corporate Governance (Module 4)

UK Corporate Governance Code (2018):

frc.org.uk/directors/corporate-governance-and-stewardship/uk-corporate-governance-code

Walker Review of Corporate Governance:

webarchive.nationalarchives.gov.uk/ukgwa/+/www.hm-treasury.gov.uk/d/walker_review_261109.pdf

Mandatory risk frameworks (Module 5)

Solvency II: bankofengland.co.uk/pra/pages/solvency2/default.aspx

ORSA: content.naic.org/cipr-topics/own-risk-and-solvency-assessment-orsa

Actuarial Profession: actuaries.org.uk/standards

Advisory risk frameworks (Module 6)

We highly recommend you take a look at The Orange Book. It can be downloaded at gov.uk/government/publications/orange-book. It is easy to read and reinforces many of the key messages from the SP9 course.

The Canadian Integrated Risk Management Framework was replaced in late 2010 with an updated version called 'The Framework for the Management of Risk'. This can be downloaded from tbs-sct.gc.ca/pol/doc-eng.aspx?id=19422

The IRM/AIRMIC/Alarm standard and other useful risk management resources can be downloaded from the IRM website at theirm.org.

Behavioural finance (Module 13)

Nigel Taylor (2000), 'Making actuaries less human: lessons in behavioural finance', Staple Inn Actuarial Society (SIAS):

sias.org.uk/media/1187/making-actuaries-less-human-lessons-from-behavioural-finance.pdf

Climate change (Modules 13 and 25)

The Prudential Regulation Authority (PRA) report entitled, "The impact of climate change on the UK insurance sector":

bankofengland.co.uk/prudential-regulation/publication/2015/the-impact-of-climate-change-on-the-uk-insurance-sector

The June 2017 recommendations of the Task Force on Climate-related Financial Disclosures (TCFD):

assets.bbhub.io/company/sites/60/2020/10/FINAL-2017-TCFD-Report-11052018.pdf

The May 2017, IFoA risk alert on climate change:

actuaries.org.uk/system/files/field/document/Risk%20Alert%20-%20Climate%20Change%20FINAL.pdf

The Bank of England's Biennial Exploratory Scenario: Financial Risks from Climate Change:

bankofengland.co.uk/stress-testing/2021/key-elements-2021-biennial-exploratory-scenario-financial-risks-climate-change

Copulas (Module 18)

Articles on copulas from *Risk* magazine:

risk.net/topics/copulas

Market Risk Management – The Greeks (Module 27)

CM2 Course Notes – The Actuarial Education Company

Credit risk management – Due diligence (Module 28)

For an extensive due diligence checklist see 'The Financial Risk Manual - A Systematic Guide to Identifying and Managing Financial Risk' J. Holliwell, ISBN-13: 9780273624189.

Operational risk management – Tokyo-Mitsubishi and NatWest (Module 29)

'Risk Management', Crouhy, Galai and Mark (page 597) ISBN-13: 978-0071357319

Capital (Module 30)

The following paper used to be, but is no longer, required reading for Subject SP9.

- *Specialty Guide on Economic Capital* – Society of Actuaries

  Version 1.5, dated March 2004

  actuaries.org.uk/documents/specialty-guide-economic-capital

X3.1 The multivariate normal distribution (MVND) is usually not a very good choice for modelling a lot of real-life risk / loss situations. The reasons for this include:

- the tails of the marginal distributions are too thin and so do not place enough weight on the probability of extreme events
- the joint tails (of the multivariate distribution) do not place enough weight on joint extreme events
- the distribution exhibits elliptical symmetry, which is unlikely to be exhibited in real-life.

Notwithstanding the above, the MVND is sometimes used within a modelling approach.

(i) Describe why this might be the case. [2]

An individual has observed information on four stocks over a trading year. Based on that data they have calculated (correctly) a coefficient of skewness of $\sqrt{1.72}$ and a kurtosis of 33.90.

(ii) Test whether the data exhibit multivariate normality by calculating Mardia's skew test statistic and Mardia's kurtosis test statistic. You should assume there are 252 trading days in a year. [4]

[Total 6]

X3.2 (i) Determine the Cholesky decomposition of the covariance matrix:

$$\mathbf{\Sigma}=\begin{pmatrix}16 & 4 & 8\\ 4 & 50 & 9\\ 8 & 9 & 54\end{pmatrix}$$ [3]

(ii) Determine the principal components of the covariance matrix:

$$\mathbf{\Sigma}=\begin{pmatrix}3 & 4\\ 4 & 9\end{pmatrix}$$ [4]

(iii) For each of (i) and (ii), state a formula for a vector, **X**, of correlated normal random variables in terms of a vector, **Z**, of independent and identically distributed standard normal random variables. [1]

[Total 8]

X3.3 (i) (a) Explain what is meant by the term 'volatility'.

(b) Give two examples of where the concept of volatility has been embedded into specific risk management activities.

[2]

(ii) State the form of an *ARCH*(1) model, the condition needed so that it models a weakly (or covariance) stationary process, and the variance of the process in this case. [3]

(iii) Discuss the strengths and weaknesses of the *ARCH*(1) model. [2]

(iv) Describe how an $ARCH(1)$ model can be fitted to data including how the parameter estimates can be obtained. [5]

[Total 12]

X3.4 An investment analyst believes that daily log returns on shares are independent and identically distributed from one time period to another. The analyst proposes the following time series model:

$$X_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = e_t$$

where:

- S_t is the share price at time t, where time is measured in days
- e_t is white noise, with mean 0 and constant variance σ^2.

(i) Describe how the fit of this model could be tested. [4]

The table below shows correlations between the daily log returns achieved by Companies X and Y over time.

Stock	**Correlation between daily log returns at each time lag**				
	Lag 0	**Lag 1**	**Lag 2**	**Lag 3**	**Lag 4**
Company X	1	0.9	0.85	0.8	0.5
Company Y	1	0.02	−0.01	0.05	0.007

(ii) Comment on the appropriateness of the model suggested above by the analyst in light of the correlation data. [3]

On further inspection of the log return data, the analyst notices that the data exhibits volatility clustering, *ie* periods of high volatility followed by periods of low volatility.

(iii) Propose, including relevant formulae, how this feature can best be allowed for in a time series model. [4]

(iv) Suggest what further correlation data the analyst should consider in order to make a decision as to whether to use a model of the type suggested in part (iii). [1]

[Total 12]

X3.5 A European company manufactures widgets that require a specialist component called a 'doohickey'. There are only two doohickey suppliers in Europe, LeHicky and DasHicky. International trade in doohickies is restricted and sourcing doohickies from outside Europe is financially punitive due to high import tariffs.

In order to protect supplies of doohickies, the company's risk policy states that it must have at least two suppliers.

Solution X3.1

Multivariate normality is discussed in Module 16 (Statistical distributions).

(i) ***Appropriateness of using multivariate normal distribution (MVND)***

The MVND may be used in a modelling approach:

- for continuity with the approach adopted previously … [½]

  … *eg* so as to isolate and facilitate an impact assessment of other (perhaps more material) potential changes to the modelling approach [½]

- for reasons of parsimony, … [½]

  … *eg* the marginal benefit of increasing the complexity (if using a different distribution) would be outweighed by the increase in costs … [½]

  … and operational (model) risks [½]

- to reduce secondary operational risks, *eg* users / beneficiaries of the model being more likely to understand the underlying assumptions. [½]

The MVND might be used as a component of the modelling scheme, with its limitations being mitigated by another part of the scheme … [½]

… *eg* the main body of the distribution used in the modelling scheme might be drawn from the MVND, but the tails of the distribution used might be based on a different distribution with fatter tails (the different elements being spliced together). [½]

[Maximum 2]

(ii) ***Testing for joint normality***

Assuming there are 252 trading days in a year, we can calculate two test statistics as follows:

Mardia's skew test statistic $=\frac{T}{6}\omega_N \ =\frac{1}{6}\times 252\times\sqrt{1.72}=55.08$ [½]

Under the null hypothesis that the data do exhibit joint normality this test statistic should follow a $\chi^2_{4(4+1)(4+2)/6}=\chi^2_{20}$ distribution. [½]

55.08 is greater than the critical point of the χ^2_{20} distribution at any measurable level and so we reject the hypothesis that the data exhibits multivariate normality. [1]

Mardia's kurtosis test statistic $=\frac{k_N - N(N+2)}{\sqrt{8N(N+2)/T}}=\frac{33.90-4\times(4+2)}{\sqrt{8\times 4\times(4+2)/252}}=11.34$ [½]

Under the null hypothesis that the data do exhibit joint normality this test statistic should follow a $N(0,1)$ distribution. [½]

11.34 is clearly greater than the critical point of the $N(0,1)$ distribution at any measurable level and so we also reject the hypothesis that the data exhibit joint normality under this test. [1]

[Total 4]

Solution X3.2

Cholesky decomposition and principal components analysis (PCA) are discussed in Module 16 (Statistical distributions).

(i) ***Cholesky decomposition***

Since $\mathbf{\Sigma}$ is a covariance matrix, it must be a positive definite matrix so it can be written in the form $\mathbf{\Sigma = CC'}$, where $\mathbf{C}$ is a lower diagonal matrix, *ie* $\mathbf{\Sigma}$ can be written as:

$$\begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{pmatrix} \text{ where } a_{ij} > 0$$

So we have:

$$\begin{pmatrix} 16 & 4 & 8 \\ 4 & 50 & 9 \\ 8 & 9 & 54 \end{pmatrix} = \begin{pmatrix} a_{11}^2 & a_{11}a_{21} & a_{11}a_{31} \\ a_{11}a_{21} & a_{21}^2 + a_{22}^2 & a_{21}a_{31} + a_{22}a_{32} \\ a_{11}a_{31} & a_{21}a_{31} + a_{22}a_{32} & a_{31}^2 + a_{32}^2 + a_{33}^2 \end{pmatrix} \quad [1]$$

And it follows that:

$$a_{11} = +\sqrt{16} = 4$$

$$a_{21} = \frac{4}{a_{11}} = 1$$

$$a_{22} = +\sqrt{50 - a_{21}^2} = +\sqrt{49} = 7$$

$$a_{31} = \frac{8}{a_{11}} = 2$$

$$a_{32} = \frac{9 - a_{21}a_{31}}{a_{22}} = 1$$

$$a_{33} = +\sqrt{54 - a_{31}^2 + a_{32}^2} = +\sqrt{49} = 7 \quad [1]$$

Therefore, the Cholesky decomposition of $\mathbf{\Sigma}$ is:

$$\begin{pmatrix} 16 & 4 & 8 \\ 4 & 50 & 9 \\ 8 & 9 & 54 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 1 & 7 & 0 \\ 2 & 1 & 7 \end{pmatrix}\begin{pmatrix} 4 & 1 & 2 \\ 0 & 7 & 1 \\ 0 & 0 & 7 \end{pmatrix} \quad [1]$$

[Total 3]

(ii) ***Principal Components Analysis***

We are trying to decompose the covariance matrix, $\mathbf{\Sigma}$, as $\mathbf{\Sigma} = \mathbf{V\Lambda V}'$ where $\mathbf{V}$ is an ($N \times N$) orthogonal matrix that consists of the (unit-length, column) eigenvectors of $\mathbf{\Sigma}$, and $\mathbf{\Lambda}$ is a diagonal matrix containing the eigenvalues of $\mathbf{\Sigma}$.

The principal components are the pairs of eigenvalues and (unit-length, column) eigenvectors of $\mathbf{\Sigma}$.

λ is an eigenvalue for a matrix $\mathbf{\Sigma}$ if it is a solution to the characteristic equation $|\mathbf{\Sigma} - \lambda \mathbf{I}| = 0$.

So we have:

$$\begin{vmatrix} 3-\lambda & 4 \\ 4 & 9-\lambda \end{vmatrix} = 0$$ [½]

And it follows that:

$$(3-\lambda)(9-\lambda) - (4 \times 4) = 0$$

$$27 - 12\lambda + \lambda^2 - 16 = 0$$

$$\lambda^2 - 12\lambda + 11 = 0$$

$$(\lambda - 11)(\lambda - 1) = 0$$ [½]

Therefore the eigenvalues are $\lambda = 11$ and $\lambda = 1$. [½]

The eigenvectors can be found by solving the following equation: $\mathbf{\Sigma v} = \lambda \mathbf{v}$.

So we have for $\lambda = 11$:

$$\begin{pmatrix} 3 & 4 \\ 4 & 9 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 11 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$ [½]

And it follows that:

$$3v_1 + 4v_2 = 11v_1$$

$$4v_1 + 9v_2 = 11v_2$$ [½]

Therefore, $v_2 = 2v_1$, and an example eigenvector is:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$ [½]

Similarly, for $\lambda = 1$, we have $v_1 = -2v_2$, and an example eigenvector is:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$ [½]

In order for $\mathbf{V}$ to be orthogonal, we need the eigenvectors to be unit-length. Therefore the eigenvectors are:

$$\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix} \text{ and } \frac{1}{\sqrt{5}}\begin{pmatrix}-2\\1\end{pmatrix}$$ [½]

[Total 4]

Therefore, the principal components decomposition of $\mathbf{\Sigma}$ is:

$$\begin{pmatrix}3 & 4\\4 & 9\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}}\\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{pmatrix}\begin{pmatrix}11 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}}\\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{pmatrix}$$

(iii) ***Vector of correlated normal random variables***

For (i), $\mathbf{X}=\boldsymbol{\alpha}+\mathbf{CZ}$, where $\boldsymbol{\alpha}$ is the vector of means and $\mathbf{C}=\begin{pmatrix}4 & 0 & 0\\1 & 7 & 0\\2 & 1 & 7\end{pmatrix}$ [½]

For (ii), $\mathbf{X}=\boldsymbol{\alpha}+\mathbf{VLZ}$, where $\mathbf{L}=\mathbf{\Lambda}^{1/2}=\begin{pmatrix}\sqrt{11} & 0\\0 & 1\end{pmatrix}$ [½]

[Total 1]

Solution X3.3

Financial time series are discussed in Module 17. Maximum likelihood estimation is also discussed in Module 19.

(i)(a) ***Explanation of volatility***

In general, volatility refers to the variability of potential outcomes. [½]

In respect of some risks it has a specific meaning; for example under market risk it is equivalent to the standard deviation of returns on investments. [½]

(i)(b) ***Examples of volatility in risk management activities***

Volatility is important as it has many applications in risk management activities, *eg*:

- articulating risk tolerance (*eg* maximum deviation from expected, Value at Risk)
- risk assessment / quantification (*eg* deviation, Value at Risk, Black-Scholes formula)
- risk control (*eg* risk limits expressed in terms of Value at Risk)
- risk optimisation (*eg* mean-variance portfolio theory).

[½ for each distinct and reasonable example, maximum 1]
[Total 2]

Solution X4.3

Different credit risk models are discussed in Module 23 (Assessment of credit risk).

Structural models can be used to value debt as well as estimate the probability of default. Credit-migration models estimate the default probability, but must be combined with estimates of bond values (or recovery rates) in order to value the debt (*eg* CreditMetrics approach). [1]

Structural models result in an intuitive, economic explanation for the probability of default, since they are based on the capital structure of a company and changes in the value of this company … [½]

… whereas credit-migration models are statistical models (*ie* a fit to past data). [½]

Structural models respond quickly to changes in a company's prospects and outlook … [½]

… because the company's share price and the volatility of the share price are used to derive default statistics, and these will rapidly reflect changing circumstances. [½]

However, because a structural model's results are sensitive to the current economic climate, the results can be volatile … [½]

… and lead to over- and under-reaction in the same way as the stock market. [½]

In particular, the model's results are heavily influenced by market sentiment in the form of share price volatility. [½]

This can lead to a vicious circle whereby increased share price volatility can lead to higher expected default rates, which can in turn lead to selling of shares, which feeds through to volatile share prices and so on. [1]

Credit-migration models are slow to adjust to changing environments … [½]

… because the credit ratings assigned to companies are typically slow to update … [½]

… so, in volatile equity markets, the credit rating and the estimated probability of default, will likely be more stable (*ie* less sensitive to market movements) than that derived using a structural model. [½]

Parameterising a structural model may be problematic as the total value of the company's assets is rarely observable or measurable, … [½]

… whereas credit-migration models do not rely on such information. [½]

The validity of credit-migration models can be undermined if, as is not uncommon, the different credit rating agencies (*eg* S&P, Moodys, Fitch) rankings of debt do not coincide. [½]

Not all organisations have obtained a (costly) credit rating. [½]

Ratings are sometimes withdrawn, *eg* if data required by the agencies for a rating has not been made available, … [½]

… although, in such circumstances, the model might simply assume that the rating transitions in accordance with the adopted transition matrix. [½]

The credit-migration approach assumes that default probabilities for each rating in each future year can be estimated. [½]

Calibrating credit-migration models requires considerable data, which has to be collected over very long periods of time. [½]

The application of structural models may be limited by unrealistic assumptions*, eg*: [½]

- there is one outstanding bond [½]
- using an average coupon and term for all debt [½]
- the value of a company follows geometric Brownian motion (*eg* Merton model).

[½ mark for any unrealistic assumption related to a specific structural model]

The credit-migration approach can be applied to companies that are not suitable for structural approach models … [½]

… *eg* companies that have no publicly traded stock, nor sovereign debt, where there is no easily-identifiable asset (which exists for most companies). [½]

However, structural models can be used for unquoted and/or unrated companies. [½]

Credit-migration models are also useful for investigating how default risk varies with the term of the liability or debt. [½]

[Maximum 10]

Solution X4.4

The question applies concepts from Module 23 (Assessment of credit risk).

(i) ***Components of default risk***

- probability of default [½]
- loss on default, which is a function of: [½]
  - exposure to the counterparty [½]
  - likely recovery in the event of default. [½]

[Total 2]

(ii) ***Credit exposure***

Each of the three derivatives has zero market value and therefore zero *current* credit risk ... [½]

... but each has the potential to generate a credit exposure over time depending on the changes in market levels, in particular UK 5-year swap rates, and the US$/£ exchange rate. [½]

For all three derivatives, a credit exposure is only generated when the derivative becomes an asset to the bank (*ie* when the market value of the derivative is much greater than zero). [½]