Subject CS2

CMP Upgrade 2024/25

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2024 CMP to make it suitable for study for the 2025 exams. It includes replacement pages and additional pages where appropriate.

Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our 2025 *Student Brochure* for more details.

We only accept the current version of assignments for marking, *ie* those published for the sessions leading to the 2025 exams. If you wish to submit your scripts for marking but only have an old version, then you can order the current assignments free of charge if you have purchased the same assignments in the same subject in a previous year, and have purchased marking for the 2025 session.

This CMP Upgrade contains:

- all significant changes to the Syllabus and Core Reading
- additional changes to the ActEd Course Notes and Assignments that will make them suitable for study for the 2025 exams.

0 Changes to the Syllabus

This section contains all the ***non-trivial*** changes to the syllabus objectives.

The following syllabus objective has been added:

3.2.2 State the Chapman–Kolmogorov equations that represent a Markov chain.

1 Changes to the Core Reading

This section contains all the ***non-trivial*** changes to the Core Reading.

Corrections to the 2024 material have been incorporated into the 2025 text and can be found at **ActEd.co.uk/coreStudyMaterial**.

Chapter 18

Section 1.1, page 5

Equation 18.2 has been amended to include an additional equivalent expression. It now reads:

$$E(Y)=\int_0^M xf(x)dx+MP(X>M)\text{, which is equal to }\int_0^M P(X>x)\,dx \tag{18.2}$$

Section 1.1, page 7

Equation 18.3 has been amended to include an additional equivalent expression. It now reads:

$$E(Z)=\int_M^\infty (x-M)f(x)dx\text{ , which is equal to }\int_M^\infty P(X>x)dx \tag{18.3}$$

2 Changes to the ActEd material

This section contains all the ***non-trivial*** changes to the ActEd text.

Chapter 18

Section 1.1, page 6

A derivation has been added to demonstrate that the new expression added to equation 18.2 is an equivalent formula for $E(Y)$. Replacement pages are given at the end of this document.

3 Changes to the X Assignments

There are no ***non-trivial*** changes to the X assignments.

Corrections to the 2024 material have been incorporated into the 2025 text and can be found at **ActEd.co.uk/coreStudyMaterial**.

4 Changes to the Y Assignments

There are no ***non-trivial*** changes to the Y assignments.

Corrections to the 2024 material have been incorporated into the 2025 text and can be found at **ActEd.co.uk/coreStudyMaterial**.

5 Changes to the Mock Exam

There are no ***non-trivial*** changes to the Mock Exam.

Corrections to the 2024 material have been incorporated into the 2025 text and can be found at **ActEd.co.uk/coreStudyMaterial**.

6 Other tuition services

In addition to the CMP, you might find the following services helpful with your study.

6.1 Study material

For further details on ActEd's study materials, please refer to the *Products* pages on the ActEd website at **ActEd.co.uk**.

6.2 Tutorials

We offer the following (face-to-face and/or online) tutorials in Subject CS2:

- Regular Tutorials (five full days / ten half days)
- Block Tutorials (five days)
- a Preparation Day for the practical exam
- An Online Classroom.

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **ActEd.co.uk**.

6.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to the 2025 *Student Brochure*, which is available from the ActEd website at **ActEd.co.uk**.

6.4 Feedback on the study material

ActEd is always pleased to receive feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course, please send them by email to **CS2@bpp.com**.

We are now in a position to consider the statistical calculations relating to reinsurance arrangements.

1.1 Excess of loss reinsurance

In excess of loss reinsurance, the insurer will pay any claim in full up to an amount M, the retention level; any amount above M will be borne by the reinsurer.

The excess of loss reinsurance arrangement can be written in the following way: if the claim is for amount X, then the insurer will pay Y where:

$$Y = X \quad \text{if } X \le M$$

$$Y = M \quad \text{if } X > M$$

The reinsurer pays the amount $Z = X - Y$.

Question

Write down an expression for Y if only a layer between M and $2M$ is reinsured.

Solution

$$Y = \begin{cases} X & \text{if } \; X \le M \\ M & \text{if } \; M < X \le 2M \\ X - M & \text{if } \; X > 2M \end{cases}$$

The insurer's liability is affected in two obvious ways by reinsurance:

(i) the mean amount paid is reduced;

(ii) the variance of the amount paid is reduced.

Both these conclusions are simple consequences of the fact that excess of loss reinsurance puts an upper limit on large claims.

The mean amounts paid by the insurer and the reinsurer under excess of loss reinsurance can now be obtained. Observe that the mean amount paid by the insurer without reinsurance is:

$$E(X) = \int_0^\infty x\, f(x)\, dx \tag{18.1}$$

where $f(x)$ is the PDF of the claim amount X. With a retention level of M the mean amount paid by the insurer becomes:

$$E(Y) = \int_0^M x f(x) dx + M P(X > M), \text{ which is equal to } \int_0^M P(X > x)\, dx \tag{18.2}$$

The first expression for the expectation can be derived as follows:

$$E(Y)=\int_0^M x\,f(x)dx+\int_M^\infty M\,f(x)dx=\int_0^M x\,f(x)dx+M\int_M^\infty f(x)dx$$

We also have:

$$\int_M^\infty f(x)dx=P(X>M)$$

Substituting this in gives the desired result.

We can calculate $E(Y^2)$ in a similar way:

$$E(Y^2)=\int_0^M x^2\,f(x)dx+\int_M^\infty M^2\,f(x)dx=\int_0^M x^2\,f(x)dx+M^2\,P\left(X>M\right)$$

Then $\text{var}(Y)=E(Y^2)-\left[E(Y)\right]^2$.

To see why $E(Y)$ is also equal to $\int_0^M P(X>x)dx$, we can rewrite the term $\int_0^M x\,f(x)dx$ as follows:

$$\int_0^M x\,f(x)dx=\int_0^M\left(\int_0^x 1\;ds\right)f(x)dx=\int_0^M\left(\int_0^x f(x)\;ds\right)dx=\int_0^M\left(\int_s^M f(x)\;dx\right)ds$$

We need to be careful with the limits in the integral in the final step. Integrating *s* between 0 and *x* and then *x* between 0 and *M* in the penultimate step means we must have $0\le s\le x\le M$. We also need to reflect this when switching the order of integration.

Performing the first integration:

$$\int_0^M\left(\int_s^M f(x)\;dx\right)ds=\int_0^M F(M)-F(s)\;ds=\int_0^M P(X>s)-P(X>M)\;ds$$

Splitting the integral we get:

$$\int_0^M P(X>s)\;ds-\int_0^M P(X>M)\;ds=\int_0^M P(X>s)\;ds-M\,P(X>M)$$

Substituting this into our expression for $E(Y)$ gives:

$$E(Y) = \int_0^M xf(x)dx + MP(X > M)$$

$$= \int_0^M P(X > s)\, ds - MP(X > M) + MP(X > M)$$

$$= \int_0^M P(X > s)\, ds$$

as required.

More generally, the moment generating function of Y , the amount paid by the insurer, is:

$$M_Y(t) = E(e^{tY}) = \int_0^M e^{tx} f(x)\, dx + e^{tM} P(X > M)$$

Here we are using the formula for the expected value of a function of a continuous random variable:

$$E(h(X)) = \int_x h(x) f(x) dx$$

with:

$$h(X) = \begin{cases} e^{tX} & \text{if } X \le M \\ e^{tM} & \text{if } X > M \end{cases}$$

Question

Suppose that claim amounts are uniformly distributed over the interval $(0, 500)$. The insurer effects individual excess of loss reinsurance with a retention limit of 375.

Calculate the expected amounts paid by the insurer and the reinsurer in respect of a single claim.

Solution

Since $X \sim U(0, 500)$, the expected gross claim amount is:

$$E(X) = \frac{500}{2} = 250$$

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