Subject CT3

Corrections to 2014 study material

Comment

This document contains details of any errors and ambiguities in the Subject CT3 study materials for the 2014 exams that have been brought to our attention. We will incorporate these changes in the study material each year. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses. If you have any such comments on this course please email them to CT3@bpp.com.

You may also find it useful to refer to the Subject CT3 Frequently Asked Questions thread on the Actuarial Discussion Forum (You can reach the forums by clicking on the "Discussion Forum" button at the top of ActEd's website, or by going to http://www.acted.co.uk/forums/.) This contains useful questions asked by students studying CT3, with answers written by ActEd's tutors.

Important note

This document was last revised significantly on 18 July 2014. The date on which any corrections have been added is noted at the start of each section.

Q&A Bank 3 Solutions 3.18, page 17 *(18 July 2014)*

There is a typo in the last equation in the solution. It should read:

$$\frac{t_{13;0.025}}{\sqrt{14}} = \frac{2.160}{\sqrt{14}} = 0.5773$$

Replacement pages can be found at the end of this document.

Solution 3.18

(i) ***Sample size needed (unknown variance)***

Using the pivotal quantity $\dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$, gives a 95% confidence interval of:

$$\bar{x} \pm t_{n-1;0.025}\frac{s}{\sqrt{n}}$$

The width of this confidence interval is $2\times t_{n-1;0.025}\dfrac{s}{\sqrt{n}}$, so we require:

$$2\times t_{n-1;0.025}\frac{8.4}{\sqrt{n}} < 10 \quad \Rightarrow \quad \frac{t_{n-1;0.025}}{\sqrt{n}} < 0.5952$$

Using trial and improvement, we get:

$$\frac{t_{13;0.025}}{\sqrt{14}} = \frac{2.160}{\sqrt{14}} = 0.5773$$

Therefore we need a sample size of at least 14 individuals.

(ii) ***Sample size needed (known variance)***

Using the pivotal quantity of $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$, gives a 95% confidence interval of:

$$\mu = \bar{X} \pm 1.96\frac{\sigma}{\sqrt{n}}$$

The width of this confidence interval is $2\times 1.96\dfrac{\sigma}{\sqrt{n}}$, so we require:

$$2\times 1.96\frac{8.4}{\sqrt{n}} < 10 \quad \Rightarrow \quad 3.29 < \sqrt{n} \quad \Rightarrow \quad n > 10.8$$

Therefore we need a sample size of at least 11 individuals.

Solution 3.19

This is a Type 2 geometric distribution, which has $E(X)=\dfrac{1-\theta}{\theta}$.

The method of moments estimator is found by:

$$E(X)=\frac{1-\hat{\theta}}{\hat{\theta}}=\bar{X} \quad\Rightarrow\quad 1-\hat{\theta}=\hat{\theta}\bar{X} \quad\Rightarrow\quad \hat{\theta}(1+\bar{X})=1 \quad\Rightarrow\quad \hat{\theta}=\frac{1}{1+\bar{X}} \qquad [2]$$

Note that since it's an estimator (rather than an estimate) we use capital letters.

The likelihood function is:

$$L(\theta)=\prod_{i=1}^{n}\theta(1-\theta)^{x_i}=\theta^n(1-\theta)^{\sum x} \qquad [1]$$

Taking logs and differentiating gives:

$$\ln L(\theta)=n\ln\theta+\left(\sum x\right)\ln(1-\theta)$$

$$\Rightarrow\quad \frac{d}{d\theta}\ln L(\theta)=\frac{n}{\theta}-\frac{\sum x}{1-\theta}$$

Setting this equal to zero in order to find the maximum gives:

$$\frac{n}{\hat{\theta}}-\frac{\sum x}{1-\hat{\theta}}=0 \quad\Rightarrow\quad \frac{n}{\hat{\theta}}=\frac{\sum x}{1-\hat{\theta}} \quad\Rightarrow\quad n-n\hat{\theta}=\hat{\theta}\sum x \quad\Rightarrow\quad 1=\hat{\theta}(1+\bar{X})$$

And hence $\hat{\theta}=\dfrac{1}{1+\bar{X}}$ as before. [2]