Subject CT4

Corrections to 2014 study material

Comment

This document contains details of any errors and ambiguities in the Subject CT4 study materials for the 2014 exams that have been brought to our attention. We will incorporate these changes in the study material each year. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses, If you have any such comments on this course please email them to CT4@bpp.com.

You may also find it useful to refer to the Subject CT4 Frequently Asked Questions thread on the Actuarial Discussion Forum (you can reach the forums by clicking on the "Discussion Forum" button at the top of ActEd's Home page). This contains useful questions asked by students studying CT4, with answers written by ActEd's tutors.

Important note

This document was produced on 4 April 2014. The date on which any subsequent corrections are added will be noted at the start of each correction.

Revision Booklet 3b

Subject CT4, April 2010, Question 11 (i)

This solution has been rewritten. It now reads as follows:

(i) *Time-homogeneous?*

The transition rates for this process take constant values. So, in general, the probability that the process will be in state j at time t, given that it is in state i at time s, depends only on the length of the time interval $t-s$.

However, there is an issue with the suspended state. This has a discontinuity when the policy has been suspended for one year because the transition rate for reinstatement then suddenly drops to zero.

So, for calculations that do not involve the suspended state, for example calculating the expected time until the first catastrophic event, it would be appropriate to model the situation using a time-homogeneous approach. However, for calculations where the suspended state may be involved, we need to allow for the sudden change in the rate.

Additional clarification:

The situation described here is really "duration-inhomogeneous", as the transition rate in the suspended state depends on how long the policy has been in that state. It's unclear whether this counts as time-homogeneous or time-inhomogeneous.

To illustrate the point, suppose we let $p_{\overline{SS}}(s,t)$ *denote the probability that the policy remains in the suspended state until time* t*, given that it was in the suspended state at time* s*.*

Now consider the probabilities $p_{\overline{SS}}(0.25, 0.75)$ *and* $p_{\overline{SS}}(0.75, 1.25)$*, which both have* $t - s = 0.5$*, so they are both probabilities of remaining suspended for another 6 months.*

Because $0.75 < 1$*, for the first probability, the one-year limit cannot have taken effect yet, so* $p_{\overline{SS}}(0.25, 0.75)$ *would equal* $e^{-0.05\times 0.5}$*, ie* $e^{-0.05(t-s)}$*. The second probability* $p_{\overline{SS}}(0.75, 1.25)$ *would equal* $e^{-0.05\times 0.5}$*, ie* $e^{-0.05(t-s)}$*, provided the policy became suspended after time 0.25 (and so we haven't hit the one-year limit yet), but it would equal zero if it became suspended before time 0.25 (because time 1.25 is more than one year later). So, even when we know the values of* s *and* t*, the second probability is not fully defined without knowing when the policy became suspended (or equivalently, the duration since suspension occurred).*

The Examiners' Report gives this numerical illustration and concludes that the model is time-inhomogeneous because some probabilities do not just depend on the value of $t - s$*.*

Note, however, that the question just asks which approach would be more appropriate for modelling the situation, which makes the question more open-ended. So, provided you argued your case satisfactorily, you should have picked up some marks whether you said it was time-homogeneous or time-inhomogeneous.

ASET

Subject CT4, April 2010, Question 11 (i)

This solution has been rewritten. It now reads as follows:

(i) ***Time-homogeneous?***

A time-homogeneous model is one in which the transition probabilities between time s and time t depend only on the length of the time "gap" $t-s$, not on the precise values of s and t. This allows us to write the transition probabilities as functions with a single argument, eg $p_{12}(5)$ is the probability, given that a process is in state 1 at a particular time, that it will be in state 2 five time units later.

The transition rates for this process take constant values. So, in general, the probability that the process will be in state j at time t, given that it is in state i at time s, depends only on the length of the time interval $t-s$.

However, there is an issue with the suspended state. This has a discontinuity when the policy has been suspended for one year because the transition rate for reinstatement then suddenly drops to zero.

So, for calculations that do not involve the suspended state, for example calculating the expected time until the first catastrophic event, it would be appropriate to model the situation using a time-homogeneous approach. However, for calculations where the suspended state may be involved, we need to allow for the sudden change in the rate.

The situation described here is really "duration-inhomogeneous", as the transition rate in the suspended state depends on how long the policy has been in that state. It's unclear whether this counts as time-homogeneous or time-inhomogeneous.

To illustrate the point, suppose we let $p_{\overline{SS}}(s,t)$ denote the probability that the policy remains in the suspended state until time t, given that it was in the suspended state at time s.
Now consider the probabilities $p_{\overline{SS}}(0.25,0.75)$ and $p_{\overline{SS}}(0.75,1.25)$, which both have $t-s=0.5$, so they are both probabilities of remaining suspended for another 6 months.

Because $0.75 < 1$*, for the first probability, the one-year limit cannot have taken effect yet, so* $p_{\overline{SS}}(0.25, 0.75)$ *would equal* $e^{-0.05\times 0.5}$*, ie* $e^{-0.05(t-s)}$*. The second probability* $p_{\overline{SS}}(0.75, 1.25)$ *would equal* $e^{-0.05\times 0.5}$*, ie* $e^{-0.05(t-s)}$*, provided the policy became suspended after time 0.25 (and so we haven't hit the one-year limit yet), but it would equal zero if it became suspended before time 0.25 (because time 1.25 is more than one year later). So, even when we know the values of* s *and* t*, the second probability is not fully defined without knowing when the policy became suspended (or equivalently, the duration since suspension occurred).*

The Examiners' Report gives this numerical illustration and concludes that the model is time-inhomogeneous because some probabilities do not just depend on the value of $t-s$*.*

Note, however, that the question just asks which approach would be more appropriate for modelling the situation, which makes the question more open-ended. So, provided you argued your case satisfactorily, you should have picked up some marks whether you said it was time-homogeneous or time-inhomogeneous.

Subject CT4, April 2010, Question 12 (added 15 April 2014)

The figures given in the question for the number of survivors at ages 58 and 59 are the wrong way round. The question should say that there are 30,795 survivors at age 58 and 30,435 survivors at age 59.

Mock Exam A

Question 4

The force of mortality $\mu = 0.05$, required to complete part (ii)(b), had been omitted from the question. The full question now reads:

In a mortality investigation, 10,000 lives aged exactly x are observed for a period of one year or until their earlier death.

(i) List the data items that you would need in order to calculate estimates of the following mortality rates, and give formulae for those estimates.

(a) The probability of a life aged exactly x dying within one year.

(b) The force of mortality over the age range $(x, x+1)$. Assume the force of mortality is constant over that period. [3]

Suppose that the true force of mortality over the age range $(x, x+1)$ has a constant value μ.

(a) Give a formula for $\tilde{\mu}$, the estimator of μ, defining the symbols in your formula, and state the asymptotic distribution of $\tilde{\mu}$.

(b) Calculate $E[V]$, the total expected waiting time over the year of age x to $x+1$ for the 10,000 lives, given that $\mu = 0.05$. [4]

[Total 7]

