Subject CT5

Corrections to 2014 study material

Comment

This document contains details of any errors and ambiguities in the Subject CT5 study materials for the 2014 exams that have been brought to our attention. We will incorporate these changes in the study material each year. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses, If you have any such comments on this course please email them to CT5@bpp.com.

You may also find it useful to refer to the Subject CT5 Frequently Asked Questions thread on the Actuarial Discussion Forum (you can reach the forums by clicking on the "Discussion Forum" button at the top of ActEd's Home page). This contains useful questions asked by students studying CT5, with answers written by ActEd's tutors.

Important note

This document was produced on 4 April 2014. The date on which any subsequent corrections are added will be noted at the start of each correction.

Revision Booklet 3

Subject CT5, April 2010, Question 11 (ii)

This question and solution is missing. However, the material is still relevant, so here is the question:

Question

Thiele's differential equation for the policy value at duration t $(t > 0)$, ${}_t\overline{V}_x$, of an immediate life annuity payable continuously at a rate of £1 per annum from age x is:

$$\frac{\partial}{\partial t}{}_t\overline{V}_x = \mu_{x+t} \times {}_t\overline{V}_x - 1 + \delta \times {}_t\overline{V}_x$$

(ii) Explain this result by general reasoning. [3]

This question when originally set was worth 8 marks in total. Part (i) of this question (for 5 marks) is no longer on the CT5 syllabus.

Solution

Consider the change in the reserves over a short time period $(t, t+h)$. The reserve at time t will earn interest at the continuous rate δ (assumed constant). The resulting amount must be used to pay for the reserve at time $t+h$ and a benefit amount h to each survivor. This calculation is accurate to $o(h)$. So we get:

$${}_t\overline{V} \times (1 + h\delta) = (1 - h\mu_{x+t})({}_{t+h}\overline{V} + h) + o(h)$$

Other terms such as the interest associated with the benefit payments are of smaller order and can be absorbed into the $o(h)$ term.

Rearranging and dividing by h gives:

$$\frac{{}_{t+h}\overline{V} - {}_t\overline{V}}{h} = \mu_{x+t} \cdot {}_{t+h}\overline{V} - 1 + \delta . {}_t\overline{V} + \frac{o(h)}{h}$$

Taking the limit as $h \to 0$ gives:

$$\frac{\partial}{\partial t}{}_t\overline{V} = \mu_{x+t} \cdot {}_{t+h}\overline{V} - 1 + \delta . {}_t\overline{V}$$

We see that the rate of change of the reserve is a function of three effects. As annuitants die, there is a release of reserve as annuity payments no longer need to be made. This happens at a rate of $\mu_{x+t} \cdot {}_{t+h}\overline{V}$. The annuity payment itself reduces the reserve at a rate of 1 per annum. And interest is earned on the reserve at a rate of $\delta . {}_t\overline{V}$ per annum. The overall rate of change of the reserve is the sum of these three components.

X Assignments

Solution X4.12

The marks for part (i) of this question did not sum correctly to 12. Here is the corrected marking schedule for the question:

(i) ***Age retirement benefit***

Let:

i be the valuation rate of interest [¼]

$v = \dfrac{1}{1+i}$ [¼]

r_x be the number of age retirements between x and $x+1$, $x \le 64$ [¼]

r_{65} be the number of age retirements at exact age 65 [¼]

l_x be the number of active lives at age x exact [¼]

All of the r_x and l_x values must come from a suitable service table. [¼]

a_x^r be the expected present value at age x of a pension of 1 *pa* payable on age retirement at age x, and payable in accordance with the scheme rules. [¼]

$\{s_x\}$ is a salary scale such that:

$$\frac{s_{x+t}}{s_x} = \frac{\text{expected salary earned in year of age } (x+t, x+t+1)}{\text{expected salary earned in year of age } (x, x+1)}$$ [½]

$z_x = \dfrac{s_{x-1} + s_{x-2} + s_{x-3}}{3}$ [¼]

Assume that age retirements before age 65 occur halfway between birthdays on average. [¼]

Note to markers: please give full credit for alternative notation provided it is clearly defined and used correctly.

Past service benefit

The member is aged exactly 26 on the valuation date and has 5 years of past service. So he is already entitled to 5/60ths of final pensionable salary when he retires. [½]

If he retires in the year of age $(y, y+1)$ for $y < 65$, we are assuming it occurs at age $y + 0.5$, so his FPS will be:

$$50{,}000 \frac{z_{y+0.5}}{s_{25.25}} \qquad [1]$$

If he retires at exact age 65, his FPS will be $50{,}000 \dfrac{z_{65}}{s_{25.25}}$. [½]

Note that we have $s_{25.25}$ in the denominator since he started to earn £50,000 on 1 April 2010, when he was aged exactly 25.25.

The expected present value of the past service benefit is:

$$\frac{5}{60} \times 50{,}000 \left[\frac{r_{26}}{l_{26}} v^{0.5} \frac{z_{26.5}}{s_{25.25}} a^r_{26.5} + \frac{r_{27}}{l_{26}} v^{1.5} \frac{z_{27.5}}{s_{25.25}} a^r_{27.5} + \cdots + \frac{r_{64}}{l_{26}} v^{38.5} \frac{z_{64.5}}{s_{25.25}} a^r_{64.5} + \frac{r_{65}}{l_{26}} v^{39} \frac{z_{65}}{s_{25.25}} a^r_{65} \right] \qquad [½]$$

$$= \frac{5}{60} \times 50{,}000 \left[\frac{r_{26}\, v^{26.5}\, z_{26.5}\, a^r_{26.5} + \cdots + r_{64}\, v^{64.5}\, z_{64.5}\, a^r_{64.5} + r_{65}\, v^{65}\, z_{65}\, a^r_{65}}{s_{25.25}\, v^{26} l_{26}} \right] \qquad [½]$$

Note that $r_x = 0$ for $x < 60$ so it would also be correct to write the expected present value of the past service liability as:

$$\frac{5}{60} \times 50{,}000 \left[\frac{r_{60}\, v^{60.5}\, z_{60.5}\, a^r_{60.5} + \cdots + r_{64}\, v^{64.5}\, z_{64.5}\, a^r_{64.5} + r_{65}\, v^{65}\, z_{65}\, a^r_{65}}{s_{25.25}\, v^{26} l_{26}} \right]$$

Now define:

$$D_x = v^x \, l_x$$ [¼]

$${}^zC_x^{ra} = r_x \, v^{x+0.5} \, z_{x+0.5} \, a_{x+0.5}^r \text{ for } x < 65$$ [¼]

$${}^zC_{65}^{ra} = r_{65} \, v^{65} \, z_{65} \, a_{65}^r$$ [¼]

and:

$${}^zM_x^{ra} = {}^zC_x^{ra} + {}^zC_{x+1}^{ra} + \cdots + {}^zC_{65}^{ra}$$ [¼]

Then the expected present value of the past service benefit is:

$$\frac{5}{60} \times 50,000 \left(\frac{{}^zC_{26}^{ra} + {}^zC_{27}^{ra} + \cdots + {}^zC_{65}^{ra}}{s_{25.25} \, D_{26}} \right) = \frac{5}{60} \times 50,000 \times \frac{{}^zM_{26}^{ra}}{s_{25.25} \, D_{26}}$$ [1]

Note that:

$$\frac{5}{60} \times 50,000 \left(\frac{{}^zC_{60}^{ra} + {}^zC_{61}^{ra} + \cdots + {}^zC_{65}^{ra}}{s_{25.25} \, D_{26}} \right) = \frac{5}{60} \times 50,000 \times \frac{{}^zM_{60}^{ra}}{s_{25.25} \, D_{26}}$$

is also correct.

Future service benefit

Consider the year of service from age y to age $y+1$. If the member completes this year of service, he will accrue 1/60th of FPS towards his annual pension. [½]

If he does not complete the year, he will accrue nothing. [½]

So the expected present value of the benefit in respect of the year of future service from age y to age $y+1$ is:

$$\frac{1}{60}\times 50,000\left[\frac{r_{y+1}}{l_{26}}v^{y+1.5-26}\frac{z_{y+1.5}}{s_{25.25}}a^r_{y+1.5}+\frac{r_{y+2}}{l_{26}}v^{y+2.5-26}\frac{z_{y+2.5}}{s_{25.25}}a^r_{y+2.5}+\cdots\right.$$ [½]

$$\left.+\frac{r_{64}}{l_{26}}v^{38.5}\frac{z_{64.5}}{s_{25.25}}a^r_{64.5}+\frac{r_{65}}{l_{26}}v^{39}\frac{z_{65}}{s_{25.25}}a^r_{65}\right]$$

$$=\frac{1}{60}\times 50,000\left[r_{y+1}\,v^{y+1.5}\,z_{y+1.5}\,a^r_{y+1.5}+\cdots\right.$$

$$\left.+r_{64}\,v^{64.5}\,z_{64.5}\,a^r_{64.5}+r_{65}\,v^{65}\,z_{65}\,a^r_{65}\right]\Big/ s_{25.25}\,v^{26}l_{26}$$ [½]

$$=\frac{1}{60}\times 50,000\left[\frac{{}^zC^{ra}_{y+1}+\cdots+{}^zC^{ra}_{65}}{s_{25.25}\,D_{26}}\right]$$ [¼]

$$=\frac{1}{60}\times 50,000\times\frac{{}^zM^{ra}_{y+1}}{s_{25.25}\,D_{26}}$$ [¼]

Note that ${}^zM^{ra}_{y+1}={}^zM^{ra}_{60}$ *for* $y\le 59$.

Now we sum over all possible years of future service that would lead to accrual. Because the pension is subject to a maximum of 40 years of accrual and the member has already accrued 5 years, we sum over the years of service (26,27), (27,28), …, (60,61). So the expected present value of the future service benefit is:

$$\frac{1}{60}\times 50,000\times\frac{{}^zM^{ra}_{27}+{}^zM^{ra}_{28}+\cdots+{}^zM^{ra}_{61}}{s_{25.25}\,D_{26}}$$ [1]

Now defining:

$$^{z}R_x^{ra} = {}^{z}M_x^{ra} + {}^{z}M_{x+1}^{ra} + \cdots + {}^{z}M_{65}^{ra}$$ [¼]

the expected present value of the future service benefit is:

$$\frac{1}{60} \times 50,000 \times \frac{{}^{z}R_{27}^{ra} - {}^{z}R_{62}^{ra}}{s_{25.25}\, D_{26}}$$ [½]

[Total 12]

Note that:

$$\frac{1}{60} \times 50,000 \times \frac{34\,{}^{z}M_{60}^{ra} + {}^{z}M_{61}^{ra}}{s_{25.25}\, D_{26}} = \frac{1}{60} \times 50,000 \times \frac{33\,{}^{z}M_{60}^{ra} + {}^{z}R_{60}^{ra} - {}^{z}R_{62}^{ra}}{s_{25.25}\, D_{26}}$$

is also correct.

Q & A Bank

Solution 5.4

Part (i) of this question uses the wrong random variable future lifetime inside the annuity function. We should be using a K and not a T. The solution now reads:

(i) ***Present value random variable***

The present value of the profit will be (making no allowance for expenses):

$$X = 8,500 - 500\, \ddot{a}_{\overline{K_{\overline{65:70}}+1}|}$$ [1]

where $K_{\overline{x:y}}$ is the future lifetime of the last survivor status, *ie* the duration until the second death occurs. [1]

[Total 2]

