Subject CT5

Corrections to 2015 study material

Comment

This document contains details of any errors and ambiguities in the Subject CT5 study materials for the 2015 exams that have been brought to our attention. We will incorporate these changes in the study material each year. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses. If you have any such comments on this course please email them to CT5@bpp.com.

You may also find it useful to refer to the Subject CT5 threads on the Actuarial Discussion Forum. (You can reach the Forum by clicking on the "Discussion Forums" button at the top of ActEd's website, or by going to www.acted.co.uk/forums/.)

Important note

This document was produced on 12 November 2014. The date on which any subsequent corrections are added will be noted at the start of each correction.

Course Notes

Chapter 5

Page 60 (added 24 November 2014)

The end-of-year reserve is incorrectly used in place of the DSAR in the calculation of the EDS and the ADS in Solution 5.17. The correct figures are as follows:

$$DSAR = 60,000 \times 1.04^{½} - {}_6V = 49,833.33$$

$$EDS = 1,900 \times q_{70} \times 49,833.33 = 2,346,537$$

$$ADS = 33 \times 49,833.33 = 1,644,500$$

So the mortality profit is £702,037.

Chapter 6

Page 12 (added 10 December 2014)

In the section on whole life assurance policies with compound increases, the description incorrectly states that the sum assured increases at the start of each year, rather than at the end of each year.

So, the sentence in the middle of the page should read:

"With this contract, the sum assured increases (by multiplying the previous sum assured by $(1+b)$) at the *end* of each year."

Page 35 (added 10 December 2014)

There is a typo in the second sentence of the second paragraph: **underling** should be **underlying**.

Chapter 10

Page 19

There is a typo in the first sentence of the penultimate paragraph. It should say:

"We need to be clear about what the Core Reading **means** by ..."

Page 29 (added 22 January 2015)

There is a typo in the Core Reading paragraph: **underling** should be **underlying**.

Page 32 (added 22 January 2015)

In the integral expression in the example, the expression $(a\mu)_{48}^{d}$ should read $(a\mu)_{48+t}^{d}$.

Page 46 (added 19 November 2014)

At the end of the first line of Solution 10.7 it should say "to age $x+t$" (and not age $x+1$).

Chapter 12

The solution to Example 3, in Section 2.3 of Chapter 12, is incorrect. This affects pages 23-26 of the current notes, as well as the solutions to Questions 12.15, 12.16 and 12.17.

Replacement pages are attached to this correction note. In order to update your notes, please:

(1) remove pages 23-26 from your current notes, and replace them with pages 23-27, which can be found at the back of this corrections document

(2) remove pages 43-46 from your current notes, and replace them with pages 43-46 again to be found after this corrections document.

Chapter 14

Page 15 (added 22 January 2015)

There are some errors of punctuation in the second Core Reading paragraph in Section 2.7. The paragraph should read:

The variation has been seen to continue after retirement but reduces at the very highest ages, although the evidence is disputed. This convergence of mortality between subgroups at higher ages is referred to as *mortality convergence*.

2.3 Example 3: Single premium unitised with-profits contract

For this example we assume the contract has a five-year term, paid for by a single premium. The allocation rate is 100% and there is no bid-offer spread. The only charges under the contract are an admin fee equal to 0.5% of the bid value of the unit fund, deducted at the end of each year by cancelling units, and a penalty on surrender calculated as a percentage of the unit fund value depending on policy duration as follows:

After 1 year: 2.4%

After 2 years: 1.8%

After 3 years: 1.2%

After 4 years: 0.6%

The unit price increases each year according to the insurer's declared bonus interest rate. The full value of the unit fund, plus any discretionary terminal bonus, is paid out on maturity or at the end of the year of earlier death. Surrender is permitted only at the end of each year, when the fund value less the above surrender penalty would be paid out.

You can assume that, over the long term, all profits earned from investment returns are fully distributed to policyholders through the bonus payments (we explain the significance of this assumption after the solution, below).

Calculate the projected profit for each policy year based on the following assumptions:

Age at entry:	45 exact
Single premium:	£20,000
Mortality:	$q_x = 0.002$ for $x = 45, 46, ..., 49$
Surrender rate:	5% of all policies in force at the end of each of Years 1-4
Bonus interest rate:	3.5% *pa*
Non-unit interest:	2% *pa*
Initial expenses:	£300
Renewal expenses:	£25 at the start of Year 2, and thereafter at the start of each subsequent year, inflating at 2.5% *pa*
Claim expenses:	£100 per death, surrender, or maturity
Non-unit reserves:	£50 per policy in force at the start of Year 5

Solution

The first thing we need to do is to project the unit fund values and the policy fees each year. At the end of the first year, the fund value (before deduction of the policy fee) is the single premium plus one year's bonus:

$$20,000 \times 1.035 = 20,700$$

The policy fee at the end of the year is:

$$20,700 \times 0.005 = 103.50$$

which means that the fund remaining at the end of the year is:

$$20,700 - 103.50 = 20,596.50$$

Working through the other years in a similar way, we obtain the following table of projected values:

Year t	Fund at end of year before deduction of policy fee (A)	Policy fee (B)	Fund at end of year after deduction of policy fee (C)
1	20,700	103.50	20,596.50
2	21,317.38	106.59	21,210.79
3	21,953.17	109.77	21,843.40
4	22,607.92	113.04	22,494.88
5	23,282.20	116.41	23,165.79

where, for years $t = 2, 3, 4, 5$:

$$(A)_t = (C)_{t-1} \times 1.035$$

$$(B) = (A) \times 0.005$$

$$(C) = (A) - (B)$$

Next we need to work out the surrender penalties. These are equal to the end-year fund values (after charges) multiplied by the appropriate percentage rates:

Year t	Fund at end of year	Surrender penalty
1	20,596.50	494.32
2	21,210.79	381.79
3	21,843.40	262.12
4	22,494.88	134.97
5	23,165.79	0

We will also need the dependent probabilities of decrement by death and surrender. Because surrenders take place at the end of each year, the dependent probability of dying is the same as the independent probability, *ie* equal to 0.002 each year. The dependent surrender probability is found as:

$$(1-0.002)\times 0.05 = 0.0499$$

as we are told that 5% of the *end* of year in-force policies surrender each year.

The dependent probability of a policy staying in force over any particular year is then:

$$1-0.002-0.0499 = 0.9481$$

(*or alternatively this can be calculated as* $(1-0.002)\times(1-0.05) = 0.9481$).

The non-unit cashflows can now be calculated, which for the first four years of the contract are as follows:

Year t	Initial and renewal Expenses (1)	Interest (2)	Policy fee (3)	Expected surrender profit (4)	Expected claim expenses (5)	Expected non-unit Cashflow (6)
1	– 300	– 6	103.5	24.67	– 5.19	– 183.02
2	– 25	– 0.5	106.59	19.05	– 5.19	94.95
3	– 25.62	– 0.51	109.77	13.08	– 5.19	91.53
4	– 26.27	– 0.53	113.04	6.73	– 5.19	87.78
5						

where:

$$(1)_t = (1)_{t-1} \times 1.025 \quad (3 \le t \le 5)$$

$$(2) = (1) \times 0.02$$

$$(3) = \{\text{fund value}\} \times 0.005$$

$$(4) = \{\text{surrender penalty}\} \times 0.0499$$

$$(5) = -100 \times (0.002 + 0.0499) \quad (1 \le t \le 4)$$

$$(6) = (1) + (2) + (3) + (4) + (5)$$

Question 12.15

Calculate the table entries for Year 5.

Having worked out the expected cashflows, we now need to calculate the expected profit for each year, taking into account the effect of the non-unit reserves.

For this policy, we are told that a non-unit reserve of 50 is required per policy in force at the start of Year 5 (and also therefore per policy in force at the *end* of Year 4); at all other times no non-unit reserve is to be held.

How will this affect the expected profit in Year 4? For a policy in force at the start of Year 4, the insurer expects a cashflow of 87.78 by the end of the year. However, the insurer now has to set aside reserves of 50 for each policy that's remained in force through to the end of the year. The probability of doing this is 0.9481 (from above).

So the expected profit for Year 4 (per policy in force at the start of the year) reduces to:

$$87.78 - 50 \times 0.9481 = 40.38$$

Question 12.16

Calculate the expected profit that will now be earned from a policy in force at the start of Year 5, including the reserve of 50 held at the start of the year.

As there are no reserves held at any time in Years 1-3, the expected profits for these years are all equal to the expected cashflows shown in the table above.

Non-unit reserves, such as in this example, are required for unit-linked and unitised with-profits policies wherever a future negative cashflow is expected.

Question 12.17

Given that there was an expected negative cashflow of 11.06, it might have seemed logical to hold the smallest reserve possible to cover this expected cost, *ie* 11.06 rather than the 50 actually held. Suggest a reason why the insurer might have used the higher figure.

What's the point of profit-testing for unitised with-profits?

As explained in an earlier chapter, the idea of a with-profits contract is to return to the policyholder the profits earned by the insurer on the policy over the policy term. One approach for unitised with-profits is to earmark the investment profits for policyholders (via the unit fund), and deduct explicit charges to cover the insurer's non-unit expenses and other costs, as in the example we have just looked at. By projecting the future non-unit profits, the insurer can check that its charges are on track to cover these outgoes.

This was why in the above example we assumed that all the investment profits were distributed to policyholders. An exam question may or may not state this assumption explicitly, but it generally would be implied, as the main point of profit testing these contracts in this way would be to "test" the adequacy of the charges in covering the non-unit liabilities.

Solution 12.11

The bid value of premiums allocated will be $2,000 \times 0.6 \times 0.95 = 1,140$.

The fund at end of year, before management charge, is $1,140 \times 1.06 = 1,208.40$.

The management charge is $1\% \times 1,208.40 = 12.08$.

The fund after deduction of management charge will be $1,208.40 - 12.08 = 1,196.32$ (or do as 99% of fund value).

Solution 12.12

The bid value of premiums allocated will be $2,000 \times 0.98 \times 0.95 = 1,862$.

The fund after allocation is $1,862 + 3,209.40 = 5,071.40$.

The fund at end of year, before management charge, is $5,071.40 \times 1.06 = 5,375.68$.

The management charge is 1% of this = 53.76.

The fund after deduction of this management charge will be:

$$5,375.68 - 53.76 = 5,321.92$$

Solution 12.13

Income to the non-unit fund comes from:

- at the start of the year, the margin between premium and cost of premiums allocated, ie $2,000 - 1,140 = 860$, and
- at the end of the year the fund management charge of 12.08 (from the unit fund calculations).

Note that interest on the non-unit fund is negative in the first year due to the effect of expenses:

$$(860 - 1,150) \times 6\% = -17.40$$

The total of all these elements is 854.68.

Solution 12.14

Non-unit profit/loss is income less outgo, *ie*:

$$854.68 - 1{,}150 - 9.54 = -304.86$$

Solution 12.15

The expenses in Year 5 are the Year 4 expenses increased by inflation at 2.5%. The amount is therefore:

$$26.27 \times 1.025 = 26.93$$

The interest is equal to the interest lost over the year due to the expenses incurred at the start of the year. Its amount is:

$$-26.93 \times 0.02 = -0.54$$

The policy fee income is read from the table, and for Year 5 this is 116.41.

As there is no surrender penalty (and no-one surrenders!) then the expected surrender profit from the surrender penalty is zero.

The claim expenses of 100 are paid out on all policies that become claims during the year. All policies in force at the start of year 5 will ultimately claim at the end of the year – either by surviving and receiving the maturity benefit, or by dying during the year and receiving the death benefit. The expected amount of claim expenses is then equal to:

$$(q_{49} + p_{49}) \times 100 = 100$$

The expected cashflow for year 5 is then calculated as:

$$\{-\text{ expenses}\} + \{\text{interest}\} + \{\text{policy fee}\} + \{\text{expected surrender profit}\} - \{\text{expected claim expenses}\}$$

$$= -26.93 - 0.54 + 116.41 - 100 = -11.06$$

The completed cashflow table now reads as follows:

Year t	Initial and renewal Expenses (1)	Interest (2)	Policy fee (3)	Expected surrender profit (4)	Expected claim expenses (5)	Expected non-unit Cashflow (6)
1	– 300	– 6	103.5	24.67	– 5.19	– 183.02
2	– 25	– 0.5	106.59	19.05	– 5.19	94.95
3	– 25.62	– 0.51	109.77	13.08	– 5.19	91.53
4	– 26.27	– 0.53	113.04	6.73	– 5.19	87.78
5	– 26.93	– 0.54	116.41	0	– 100	– 11.06

Solution 12.16

The expected profit for Year 5 will equal:

$$\{\text{expected cashflow}\}+\{\text{reserve at start of year}\}+\{\text{interest on reserve}\}$$
$$-\{\text{expected cost of reserve at end of year}\}$$

Putting in the values (and noting that the reserve at the end of the year is zero) we obtain:

$$-11.06+50+0.02\times 50-0=39.94$$

Solution 12.17

All reserves need to be prudent, so that there will be a high probability that the liabilities (future outgo) will be covered. The insurer would therefore have projected its cashflows on more cautious assumptions, leading to a somewhat higher negative cashflow than 11.06 in Year 5, and decided that a reserve of 50 was necessary to cover this.

Solution 12.18

£186.97 will be the profit expected by the end of the coming year (Year 2), from a policy that was in force at exact time 1.

Solution 12.19

From the definition given in Section 2.1, the required probability is:

$$ {}_1(ap)_{55} = 1 - (aq)^d_{55} - (aq)^w_{55} = 1 - 0.005 - 0.1 = 0.895 $$

So:

$$ E\left(P_2\right) = 186.97 \times {}_1(ap)_{55} = 186.97 \times 0.895 = 167.34 $$

Solution 12.20

The first-year element of the profit signature is unchanged from the profit vector, because the amount (-803.99) is already the amount per policy in force at the beginning of year 1, *ie* at inception.

For year 3 we need:

$$ (PS)_3 = (PRO)_3 \times {}_2(ap)_{55} $$

and in general we will need:

$$ (PS)_t = (PRO)_t \times {}_{t-1}(ap)_{55} $$

We calculate $(ap)_{55}$, $(ap)_{56}$, *etc* from:

$$ (ap)_x = 1 - (aq)^d_x - (aq)^w_x $$

(using the dependent probabilities given in Section 2.1); and ${}_2(ap)_{55}$ (for example) from the cumulative probability:

$$ {}_2(ap)_{55} = (ap)_{55}\,(ap)_{56} $$