Subject CT8

Corrections to 2015 study material

Comment

This document contains details of any errors and ambiguities in the Subject CT8 study materials for the 2015 exams that have been brought to our attention. We will incorporate these changes in the study material each year. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses, If you have any such comments on this course please email them to CT8@bpp.com.

You may also find it useful to refer to the Subject CT8 Frequently Asked Questions thread on the Actuarial Discussion Forum (you can reach the forums by clicking on the "Discussion Forum" button at the top of ActEd's Home page). This contains useful questions asked by students studying CT8, with answers written by ActEd's tutors.

Important note

This document was produced on 9 October 2014. The dates on which any subsequent corrections have been added are noted below.

Course Notes

Chapter 2, pages 37-38 (2 April 2015)

The solution to the exam-style question at the end of Chapter 2 has been rewritten with the expected returns and variances in decimal form rather than in % and %% units respectively. This ensures there is no inconsistency between the units of the two terms in the utility function, which would affect the value of α. The revised solution is shown on the next page.

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Solution

This is Subject 109, April 2003, Question 7.

(i) ***Maximising the investor's expected utility***

Assuming that all of the investor's money is invested, and hence the portfolio weights sum to one, the expected return and variance of a portfolio consisting of a proportion x_A of wealth held in Asset A, and a proportion $1-x_A$ of wealth held in Asset B are:

$$\begin{aligned} E_P &= x_A E_A + (1-x_A)E_B \\ &= 0.06x_A + 0.08(1-x_A) \\ &= 0.08 - 0.02x_A \end{aligned}$$

and:

$$\begin{aligned} V_P &= {x_A}^2 V_A + {x_B}^2 V_B + 2x_A x_B \sigma_A \sigma_B \rho_{AB} \\ &= 0.0004{x_A}^2 + 0.0025(1-x_A)^2 + 0.0010x_A(1-x_A) \\ &= 0.0019x_A^2 - 0.0040x_A + 0.0025 \end{aligned}$$

Therefore the investor's expected utility is:

$$\begin{aligned} E_\alpha(U) &= E(r_p) - \alpha Var(r_p) \\ &= 0.08 - 0.02x_A - \alpha\left(0.0019x_A^2 - 0.0040x_A + 0.0025\right) \end{aligned}$$

We can maximise this function of x_A by differentiating and setting to zero:

$$\frac{dE}{dx_A} = -0.02 - \alpha\left(0.0038x_A - 0.0040\right) = 0$$

$$\Leftrightarrow \qquad x_A = \frac{20\alpha - 100}{19\alpha}$$

or:

$$x_A = \frac{20}{19} - \frac{100}{19\alpha}$$

NB The second-order derivative is:

$$\frac{d^2E}{dx_A^2} = -0.0038\alpha < 0$$

which confirms that we have a maximum.

(ii) ***Show that the investor selects an increasing proportion of Asset A***

Differentiating the formula for the optimal value of x_A in terms of α gives:

$$\frac{dx_A}{d\alpha} = \frac{100}{19\alpha^2} > 0$$

This confirms that as α increases, so x_A, the proportion of wealth held in Asset A, increases too.

Chapter 6, page 10 (11 June 2015)

There are errors in the formulae used to estimate beta and alpha in the middle of this page. They should say:

$$\hat{\beta}_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\sum_{t=1}^{N}[(R_{it} - \bar{R}_i)(R_{Mt} - \bar{R}_M)]}{\sum_{t=1}^{N}(R_{Mt} - \bar{R}_M)^2}$$

$$\hat{\alpha}_i = \bar{R}_i - \hat{\beta}_i \bar{R}_M$$

In addition, the following line should say:

where $\bar{R}_i = \frac{1}{N}\sum_{t=1}^{N} R_{it}$ and $\bar{R}_M = \frac{1}{N}\sum_{t=1}^{N} R_{Mt}$

Chapter 13, page 48 (31 July 2015)

The first number in the bracket at the top of this page should be 1,040.81 and not 1,041.81.

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Chapter 16, page 39

There is a typo in the third line of the solution to Question 16.9. The formula for D_t is missing an S_0 and should say $D_t = S_0 \, e^{X_t}$

.

Q&A Bank

Solution 5.7 (ii)

There is a typo in the notation used in the final line at the bottom of page 14. It should say $V_1(2) = 65.568$.

Solution 5.9 (iv)

There is a typo in the formula at the top of page 19. The "2" should be deleted, so that the formula reads:

$$\sum_{i=1}^{n} x_i^2 C_{ii} + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} x_i x_j C_{ij}$$

The differentiation with respect to x_i in the next line is correct and still gives:

$$2x_i C_{ii} + 2\sum_{\substack{j=1 \\ j \neq i}}^{n} x_j C_{ij} = 2\sum_{j=1}^{n} x_j C_{ij}$$

since the second term will give a contribution from each summation.

Revision Notes – Booklet 2 (19 March 2015)

Solution to Past Exam Question 3 – Subject CT8, April 2005, Question 4

The solution to part (ii) has been changed and expanded to:

(ii) ***How many parameters?***

In the multifactor model, we have:

$$R_i = a_i + b_{i,1}I_1 + \cdots + b_{i,L}I_L + c_i$$

where:

- $E\left[c_i\right] = 0$ for $i = 1...n$
- $\text{cov}\left[c_i, c_j\right] = 0$ for $i = 1...n,\ j = 1...n,\ i \neq j$
- $\text{cov}\left[I_k, c_i\right] = 0$ for $i = 1...n,\ k = 1...L$

It is possible to set the L factors so that they are orthogonal and hence uncorrelated with each other, *ie* $\text{cov}\left[I_k, I_l\right] = 0$ for $k = 1...L,\ l = 1...L,\ k \neq l$.

Assuming a portfolio of N securities, in order to calculate the mean and the variance of the return on this portfolio, we would need the following data items:

- N lots of a_i's
- NL lots of $b_{i,k}$'s
- N variances $\text{var}\left[c_i\right]$
- L expectations $E\left[I_k\right]$ and L variances $\text{var}\left[I_k\right]$

This gives a total of $N(L+2)+2L$ parameters.

Note that the solution in the Examiners' Report does not assume orthogonality of the I_k's and hence there would be an extra $L(L-1)/2$ covariance pairs to add to the total.

In the single index model, there is just one factor $I_1 = R_M$. In order to calculate the mean and the variance of the return on this portfolio, we would need the following data items:

- N lots of a_i's

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- N lots of $b_{i,k}$'s
- N variances $\text{var}\left[c_i\right]$
- $E\left[R_M\right]$ and $\text{var}\left[R_M\right]$

This gives a total of $3N+2$ data items.

Revision Notes – Booklet 3 (19 March 2015)

Factsheet – page 131

Ito's Lemma in Section 8 is missing a Y_t^2 from the last term in the curly brackets and should read:

Let $dX_t = A_t dt + Y_t dB_t$. If $f(t, X_t)$ is twice partially differentiable with respect to X_t and once with respect to t, then:

$$df\left(t, X_t\right) = \left\{\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} A_t + \tfrac{1}{2} Y_t^2 \frac{\partial^2 f}{\partial x^2}\right\} dt + \frac{\partial f}{\partial x} Y_t dB_t$$

Revision Notes – Booklet 4 (19 March 2015)

Factsheet – page 152

The definition of gamma in Section 9 should read:

$$\text{Gamma:} \quad \Gamma = \frac{\partial^2 f}{\partial S_t^2} = \frac{\partial \Delta}{\partial S_t}$$

Revision Notes – Booklet 5 (16 February 2015)

Solution to Past Exam Question 11 – Subject CT8, April 2008, Question 11

In the solution to part (i)(a) on page 110 of this booklet, the sign of the "discounted K" term is incorrectly shown as positive in the first formula for $\frac{\partial f}{\partial S_t}$. This formula should read:

$$\frac{\partial f}{\partial S_t} = \Phi(d_1) + S_t \phi(d_1)\frac{\partial d_1}{\partial S_t} - Ke^{-r(T-t)}\phi(d_2)\frac{\partial d_2}{\partial S_t}$$

The rest of the solution is shown correctly.

Revision Notes – Booklet 6 (2 April 2015)

Question analysis – Full solutions – page 38

In the final bullet at the bottom of the page, the $(T-t)$ should be squared. The correct partial derivative is therefore:

- $$\frac{\partial B(t,T)}{\partial t} = \left(r_r - ½\sigma^2 (T-t)^2\right)B(t,T)$$

This was a typo affecting just that line and so the rest of the solution is correct.

Mock Exam A (10 June 2015)

Solution to Question 9(i) – page 27

Under the *disadvantages* of the Hull & White model, the solution currently says:

- does not produce a *good fit to historical data* with realistic parameters values [½]

This point should in fact be listed under the *advantages* and say:

- can produce a *good fit to historical data* [½]

as the model can produce a good fit if the $\mu(t)$ function has a sufficient number of parameters.

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