Subject ST9

Corrections to 2015 study material

Comment This document contains details of any errors and ambiguities in the Subject ST9 study materials for the 2014 exams that have been brought to our attention. We will incorporate these changes in the study material each year. We are always happy to receive feedback from students, particularly details concerning any errors, contradictions or unclear statements in the courses. If you have any such comments on this course please email them to ST9@bpp.com. You may also find it useful to refer to the Subject ST9 Frequently Asked Questions thread on the Actuarial Discussion Forum. (You can reach the Forums by clicking on the "Discussion Forum" button in the bottom left-hand corner of ActEd's website, or by going to www.acted.co.uk/forums/.) This contains useful questions asked by students studying Subject ST9, with answers written by ActEd's tutors.

Important note

This document was created in January 2015. The date on which any subsequent corrections have been added is noted below.

Sweeting

Students are advised to check the errata to the Sweeting textbook. This can be found on the ActEd's ST9 online discussion forum at:

http://www.acted.co.uk/forums/forumdisplay.php?f=79

Chapter 13

Page 6 and page 15

The term α_0 should not appear in the formula within the final paragraph on page 6. It should read:

For the $AR(p)$ process described above to be stationary, the length of the p-dimensional vector containing the roots (z) of the following polynomial expression must be greater than 1:

$$f(z) = 1 - \alpha_1 z - \alpha_2 z^2 - \ldots - \alpha_p z^p = 0$$

For example, if $p = 1$ the relevant polynomial expression is:

$$f(z) = 1 - \alpha_1 z = 0$$

the root of which is given by:

$$z = \frac{1}{\alpha_1}$$

The stated requirement for covariance stationarity is that the root lies "outside the unit circle". For $p = 1$ this will be if $|\alpha_1| < 1$.

This error also appears in Sweeting on page 289 (formula 13.7) and on page 15 of Chapter 13.

Chapter 15

Page 7

To ensure the MLE is a matrix, this should read:

where: column vector $\boldsymbol{\Phi}_t^{-1} = \left(\Phi^{-1}\left(F\left(x_{1,t}\right)\right), \Phi^{-1}\left(F\left(x_{2,t}\right)\right), \ldots, \Phi^{-1}\left(F\left(x_{N,t}\right)\right)\right)'$

Chapter 16

Page 7

The standardisation 'constants' (α and β) referred to in Section 3.1 are typically sequences of constants, ie $\alpha_1,\dots,\alpha_n$ and $\beta_1,\dots,\beta_n$. This can be seen in Example 1, where:

$$\alpha_n = \frac{1}{\lambda}\ln n$$

however, in the Course Notes the subscript to α has been omitted in error. This section should read:

We can standardise this for a sequence of real constants $\beta_1,\dots,\beta_n > 0$ and $\alpha_1,\dots,\alpha_n$, and consider the limit as n increases:

$$\lim_{n\to\infty} P\left(\frac{X_M - \alpha_n}{\beta_n} \le x\right) = \lim_{n\to\infty} F^n\left(\beta_n x + \alpha_n\right)$$

This result applies for all commonly used statistical distributions from which the X_i may originate.

Example 1 – block maxima of the exponential distribution

The exponential distribution has the distribution function $F(x) = 1 - e^{-\lambda x}$. If we set $\beta = \frac{1}{\lambda}$ (*ie* $\beta_n = \beta$ for all n) and $\alpha_n = \frac{1}{\lambda}\ln n$ then we have:

$$\beta x + \alpha_n = \frac{x}{\lambda} + \frac{\ln n}{\lambda}$$

Hence:

$$\begin{aligned} F\left(\beta x + \alpha_n\right) &= F\left(\frac{x + \ln n}{\lambda}\right) \\ &= 1 - e^{-\lambda\left(\frac{x+\ln n}{\lambda}\right)} \\ &= 1 - \frac{1}{n}e^{-x} \end{aligned}$$

Therefore:

$$F^n(\beta x + \alpha_n) = \left(1 - \frac{1}{n}e^{-x}\right)^n \text{, where } x \geq -\ln n$$

and so, since $e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$ by definition, the distribution of the maxima is:

$$\lim_{n\to\infty} F^n(\beta x + \alpha_n) = e^{-e^{-x}}.$$

Chapter 16

Page 5

In the light of the correction above (for Page 7) the corresponding section on Page 5 should read:

One approach is to look at $X_M = \max(X_1, X_2, \ldots, X_n)$, the maximum value in a set of n losses – referred to as a *block maximum*. If we look at a number of such blocks then these maxima can be standardised in a similar way, *ie* we calculate expressions of the form $\frac{X_M - \alpha_n}{\beta_n}$. These standardised values can be approximated by a particular type of distribution – an *extreme value distribution*.

Chapter 12 (19/01/15)

Page 22

The CDF given is the 2-parameter version (not 3-parameter as stated). It is effectively the standardised version missing the location parameter, often given as μ.

For the 3-parameter version replace x with $(x-\mu)$. The mean is then $\mu + \frac{\beta\gamma}{\gamma - 1}$.

Q&A 3.9 (19/01/15)

In the question part (ii), *X* should be given as $X \sim Gamma(\beta = 0.05, \gamma = 5)$.